

What to expect – an R vignette for **expectreg**

Fabian Sobotka, Thomas Kneib, Sabine Schnabel, Paul Eilers

May 9, 2011

Abstract

expectreg is an R package for estimating expectile curves from univariate and multivariate data. Expectile curves are a valuable least squares alternative to quantile regression which is based on linear programming techniques. **expectreg** provides a number of functions for different approaches taken to estimate expectiles investigated since their introduction in [NEWBY and POWELL(1987)] using asymmetric least squares.

1 Overview

This section offers an overview over the functions implemented in **expectreg**. It assumes that the user already installed the package successfully.

```
> library(expectreg)

> help(package = "expectreg")
> data(package = "expectreg")
```

will give you a short overview about the available help files of the package as well as the data that will be provided with **expectreg**. The package includes the following functions:

| | |
|-----------------------------------|--|
| <code>base</code> | Creates bases for a regression based on covariates |
| <code>dkoenker</code> | Density of a special distribution developed by Roger Koenker [KOENKER(1992)] |
| <code>ebeta</code> | Expectiles of the beta distribution |
| <code>ekoenker</code> | Expectiles of a special distribution developed by Roger Koenker |
| <code>enorm</code> | Expectiles of the normal distribution |
| <code>eunif</code> | Expectiles of the uniform distribution |
| <code>expectile.boost</code> | Expectile regression using boosting |
| <code>expectile.laws</code> | Expectiles regression of additive models |
| <code>expectile.restricted</code> | Algorithm proposed by [HE(1997)] for restricted regression quantiles |
| <code>expectile.bundle</code> | Location-scale type model for non-crossing expectiles |
| <code>pkoenker</code> | Distribution function for a special distribution developed by Roger Koenker |
| <code>qkoenker</code> | Quantile function for a special distribution developed by Roger Koenker |
| <code>quant.boost</code> | Quantile regression using boosting |
| <code>rkoenker</code> | Random variable generated from a special distribution developed by Roger Koenker |

2 Expectiles in a nutshell

2.1 Introduction to expectiles using LAWS

Asymmetric least squares or least asymmetrically weighted squares (LAWS) is a weighted generalization of ordinary least squares (OLS) estimation. LAWS minimizes

$$S = \sum_{i=1}^n w_i(p) (y_i - \mu_i(p))^2,$$

with

$$w_i(p) = \begin{cases} p & \text{if } y_i > \mu_i(p) \\ 1 - p & \text{if } y_i \leq \mu_i(p) \end{cases}, \quad (1)$$

where y_i is the response and $\mu_i(p)$ is the population expectile for different values of an asymmetry parameter p with $0 < p < 1$. The model is fitted by alternating between weighted regression and recomputing weights until convergence (when the weights do not change anymore). Equal weights ($p = 0.5$) give a convenient starting point.

For the expectile curve $\mu(p)$ several choices for the functional form are possible. The original proposal in [NEWBY and POWELL(1987)] favored a linear model. We suggest a more flexible functional form for the expectile curve. [SCHNABEL and EILERS(2009)] proposed to model expectile curves with P -splines. Other types such as other splines, markov random field or other options are also possible (see [SOBOTKA and KNEIB(2010)]).

2.2 Expectile bundle model

In theory it is not possible that expectile curves cross, but in estimation practise it is often encountered due to sampling variation. The expectile bundle model is a location-scale type of model that allows for the simultaneous estimation of a set of expectiles. By its construction crossing over of curves is not possible.

In the expectile bundle model the expectiles $\mu(x, p)$ are defined by

$$\mu(x, p) = t(x) + c(p)s(x) \quad (2)$$

where $t(x)$ is a common smooth trend of all expectile curves specified by a P -spline. $c(p)$ is the asymmetry function of the bundle describing the spread, i.e. the set of standardized expectiles. $s(x)$ represents the local width of the expectile bundle and is also formulated as a P -spline. The estimation procedure consists of two steps. In Step 1 the common trend $t(x)$ is estimated. Then in step 2 we use the detrended response $y - t(x)$ to estimate $s(x)$ and $c(p)$ in an iterative procedure.

The expectiles bundle model is explained in more detail in [SCHNABEL and EILERS(2010)].

2.3 Restricted regression quantiles

In [HE(1997)] proposed a version of restricted regression quantiles to avoid the crossing of quantile curves. His model for computing non-parametric conditional quantile functions takes the following form

$$y = f(x) + s(x)e.$$

[HE(1997)] takes a three-step procedure where he determines first the conditional median function and then in a second step estimate the smooth non-negative amplitude function. The third step consists of the step wise calculation of the ‘‘asymmetry factor’’ c_α for each α -quantile curve separately.

2.4 Expectile and quantile estimation using boosting

1. Initialize all model components as $\hat{f}_j^{[0]}(\mathbf{z}) \equiv \mathbf{0}$, $j = 1, \dots, r$. Set the iteration index to $m = 1$.
2. Compute the current negative gradient vector \mathbf{u} with elements

$$u_i = - \left. \frac{\partial}{\partial \eta} \rho(y_i, \eta) \right|_{\eta = \hat{\eta}^{[m-1]}(\mathbf{z}_i)}, \quad i = 1, \dots, n.$$

3. Choose the base-learner \mathbf{g}_{j^*} that minimizes the L_2 -loss, i.e. the best-fitting function according to

$$j^* = \arg \min_{1 \leq j \leq r} \sum_{i=1}^n (u_i - \hat{g}_j(\mathbf{z})_i)^2$$

where $\hat{\mathbf{g}}_j = \mathbf{S}_j \mathbf{u}$.

4. Update the corresponding function estimate to $\hat{f}_{j^*}^{[m]} = \hat{f}_{j^*}^{[m-1]} + \nu \hat{g}_{j^*}$, where $\nu \in (0, 1]$ is a step size. For all remaining functions set $\hat{f}_j^{[m]} = \hat{f}_j^{[m-1]}$, $j \neq j^*$.
5. Increase m by one. If $m < m_{\text{stop}}$ go back to step 2., otherwise terminate the algorithm.

For expectile regression, the empirical risk is given the asymmetric least squares criterion (1) and the appropriate loss function is defined as $\rho(y, \eta) = w(\tau)(y - \eta_\tau)^2$. The corresponding negative gradient is therefore obtained as

$$u_i = 2w_i(\tau)(y_i - \eta_i).$$

3 Example and available data

Expectile estimation can be used in almost any type of situation where one is interested in estimating smooth curves in non-central parts of the data under consideration. The data provided with the package are

```
> data(india)
> data(dutchboys)
```

`india` consists of a data sample of 4000 observations with 6 variables from a 'Demographic and Health Survey' about malnutrition of children in India. Data set only contains 1/10 of the observations and some basic variables to enable first analyses. Details are given in [FENSKE *et al.*(2009)]. `dutchboys` contains data from the Fourth Dutch growth study and includes 6848 observations on 10 variables. More information can be found in [VAN BUUREN and FREDRIKS(2001)].

3.1 Basic examples

The basic function `expectile.laws` can be used to estimate 11 expectiles curves for different levels of asymmetry parameter p . The results are shown in the following graph.

```
> data(dutchboys)
> exp.l <- expectile.laws(dutchboys[, 3] ~ base(dutchboys[, 2],
+       "pspline"), smooth = "acv")
```

Due to the large number of observations in the data set crossing of curves is already unlikely to happen. Nevertheless we apply also the expectile bundle model implemented in `expectile.bundle` to this example.

```
> exp.b <- expectile.bundle(dutchboys[, 3] ~ base(dutchboys[, 2],
+       "pspline"), smooth = "none")
```

Additionally we analyze the data with the algorithm proposed in [HE(1997)] implemented in `expectile.restricted`.

```
> exp.r <- expectile.restricted(dutchboys[, 3] ~ base(dutchboys[,
+       2], "pspline"), smooth = "schall")
```

3.2 Applied boosting

```
> exp.boost <- expectile.boost(hgt ~ bbs(age, df = 5, degree = 2),
+       dutchboys, mstop = rep(500, 11))
```

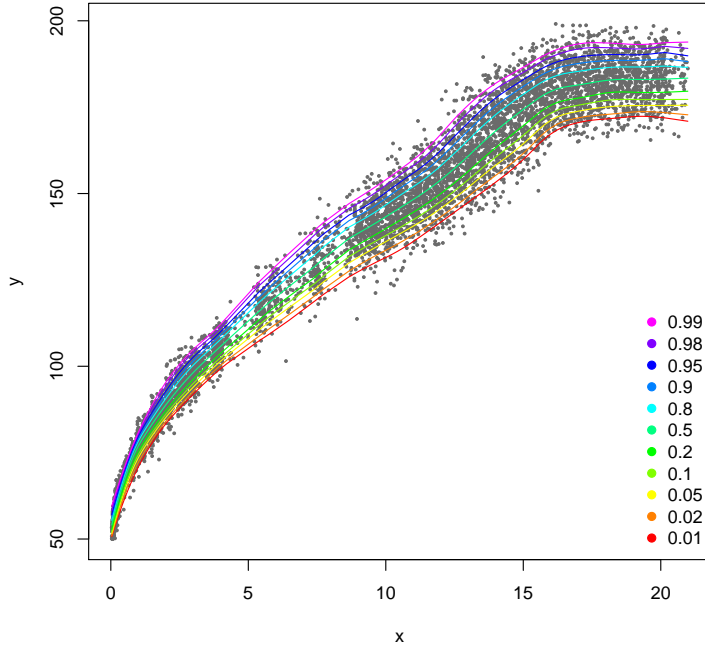


Figure 1: Expectile curves estimated using `expectile.laws`

References

- [VAN BUUREN and FREDRIKS(2001)] VAN BUUREN, S., and A. M. FREDRIKS, 2001 Worm plot: A simple diagnostic device for modeling growth reference curves. *Statistics in Medicine* **20**: 1259–1277.
- [FENSKE *et al.*(2009)] FENSKE, N., T. KNEIB, and T. HOTHORN, 2009 Identifying risk factors for severe childhood malnutrition by boosting additive quantile regression. Technical Report 52, University of Munich.
- [HE(1997)] HE, X., 1997 Quantile curves without crossing. *The American Statistician* **51**: 186–192.
- [KOENKER(1992)] KOENKER, R., 1992 When are expectiles percentiles? (solution). *Economic Theory* **9**: 526–527.
- [NEWY and POWELL(1987)] NEWY, W. K., and J. L. POWELL, 1987 Asymmetric least squares estimation and testing. *Econometrica* **55**: 819–847.
- [SCHNABEL and EILERS(2009)] SCHNABEL, S. K., and P. H. C. EILERS, 2009b Optimal expectile smoothing. *Computational Statistics and Data Analysis* **53**: 4168–4177.
- [SCHNABEL and EILERS(2010)] SCHNABEL, S. K., and P. H. C. EILERS, 2010 Non crossing expectiles and quantiles. *Journal for Computational and Graphical Statistics* (Submitted).
- [SOBOTKA and KNEIB(2010)] SOBOTKA, F., and T. KNEIB, 2010 Geoadditive Expectile Regression. *Computational Statistics and Data Analysis*, doi: 10.1016/j.csda.2010.11.015.

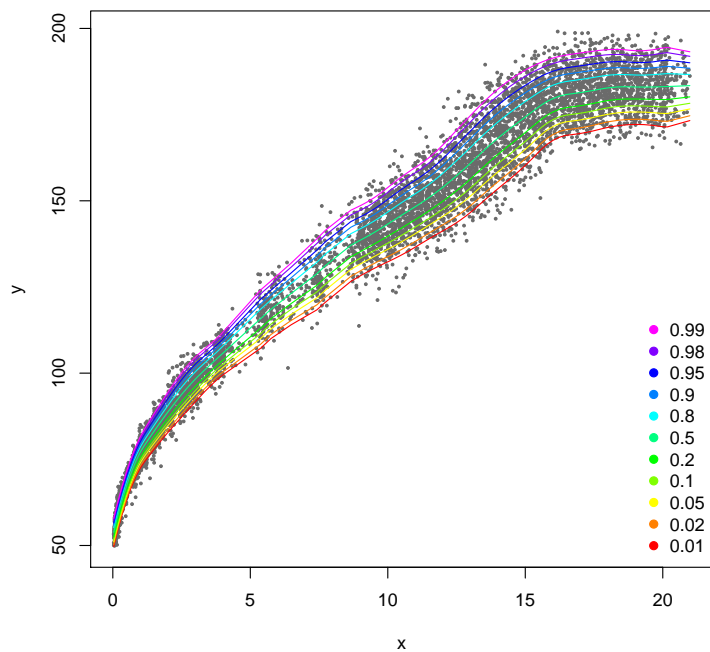


Figure 2: Expectile curves estimated using `expectile.bundle`

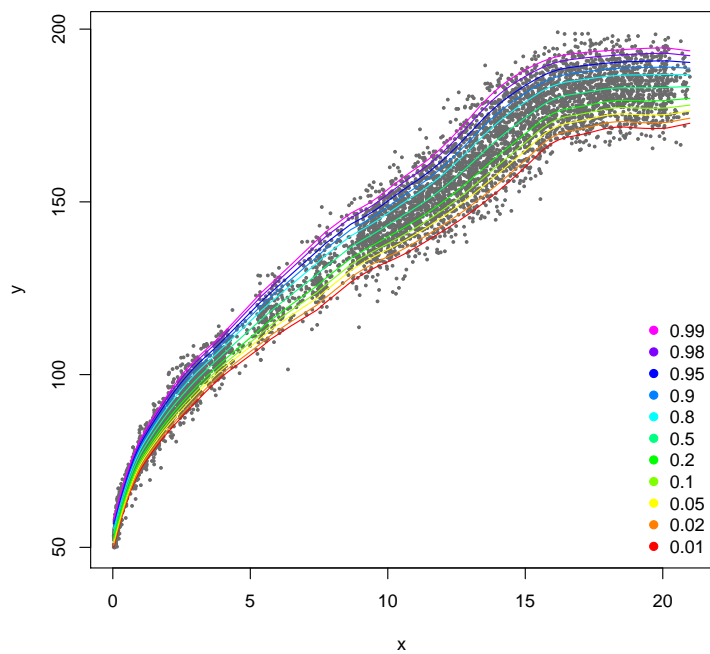


Figure 3: Expectile curves estimated using `expectile.restricted`

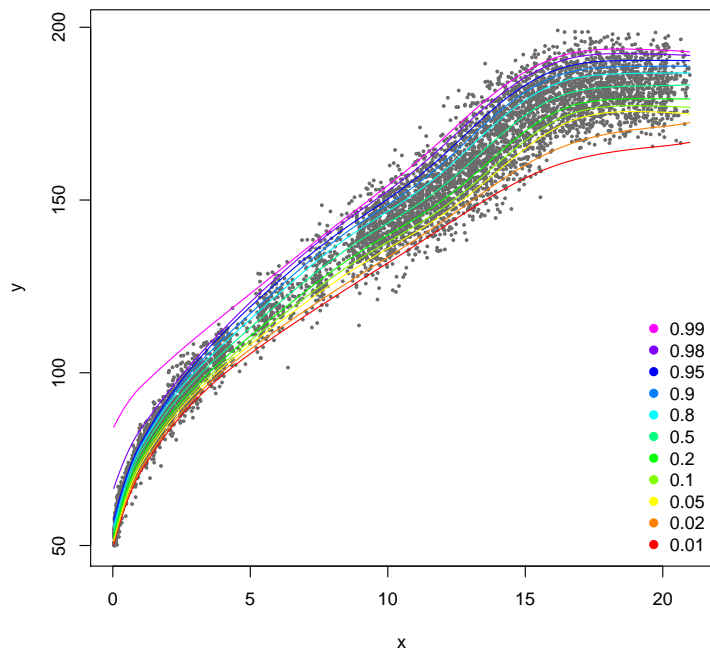


Figure 4: Expectile curves estimated using `expectile.boost`