

# Robust Generalized Linear Models

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The CC-family contains functions of composite of concave and convex functions. The CC-estimators are derived from minimizing loss functions in the CC-family via composite optimization by conjugation operator (COCO). The CC-estimators include robust generalized linear models. See Wang (2020).

## Robust logistic regression

In a UK hospital, 135 expectant mothers were surveyed on the decision of breastfeeding their babies or not, along with two-level predictive factors. Description and references can be found in Heritier et al. (2009).

```
require("mpath")
data(breastfeed)
```

Remove rows with missing values.

```
breastfeed=na.omit(breastfeed)
```

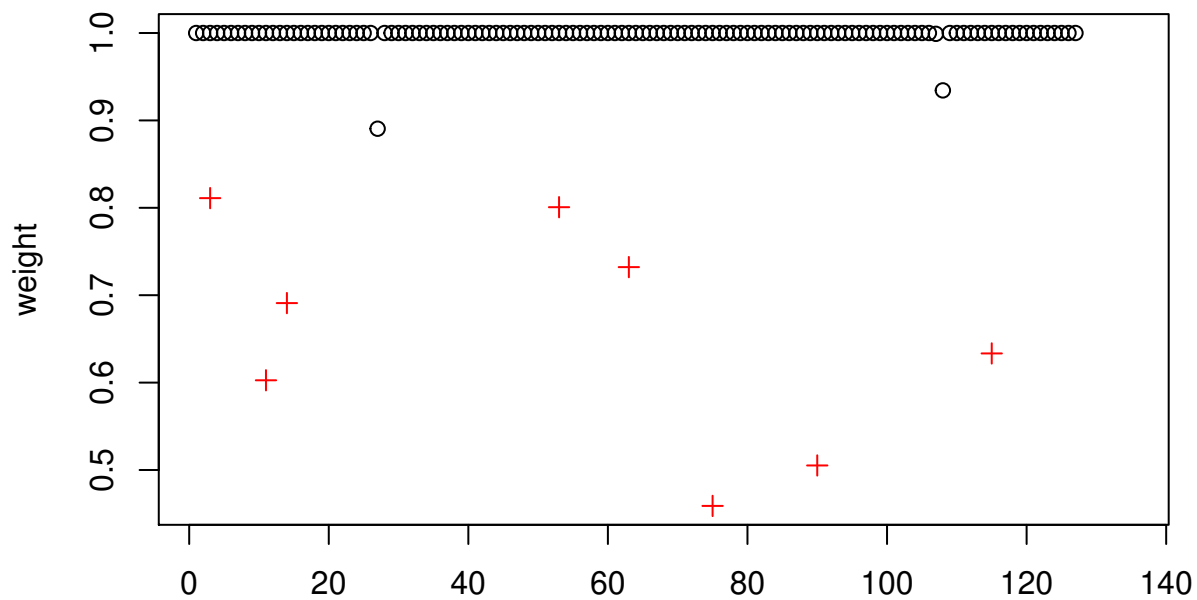
We compute binomial-induced CC-estimators, i.e., robust logistic regression, and display the robust weights for each model.

```
sval <- c(1.5, 1.5, 5, 2.5, 3.5, 2.5, 2.2, 7)
cfun <- c("hcave", "acave", "bcave", "ccave", "dcave", "gcave", "tcave", "ecave")
id <- 1:8
for(i in c(1:5,8,6,7)){
  fitnew <- ccglm(breast~., data=breastfeed, s=sval[i], cfun=i, dfun=binomial(),
    trace=FALSE)
  goodid <- sort.list(fitnew$weights_update)[id]
  plot(fitnew$weights_update, type="n", ylab="weight",
    main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")"))),
      list(cfun=cfun[i], s = sval[i]))))
  points(fitnew$weights_update[-goodid], ylab="weight",
    main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")"))),
      list(cfun=cfun[i], s = sval[i]))))
  points(sort.list(fitnew$weights_update)[id], sort(fitnew$weights_update)[id], pch=3,
    col="red")
}
```

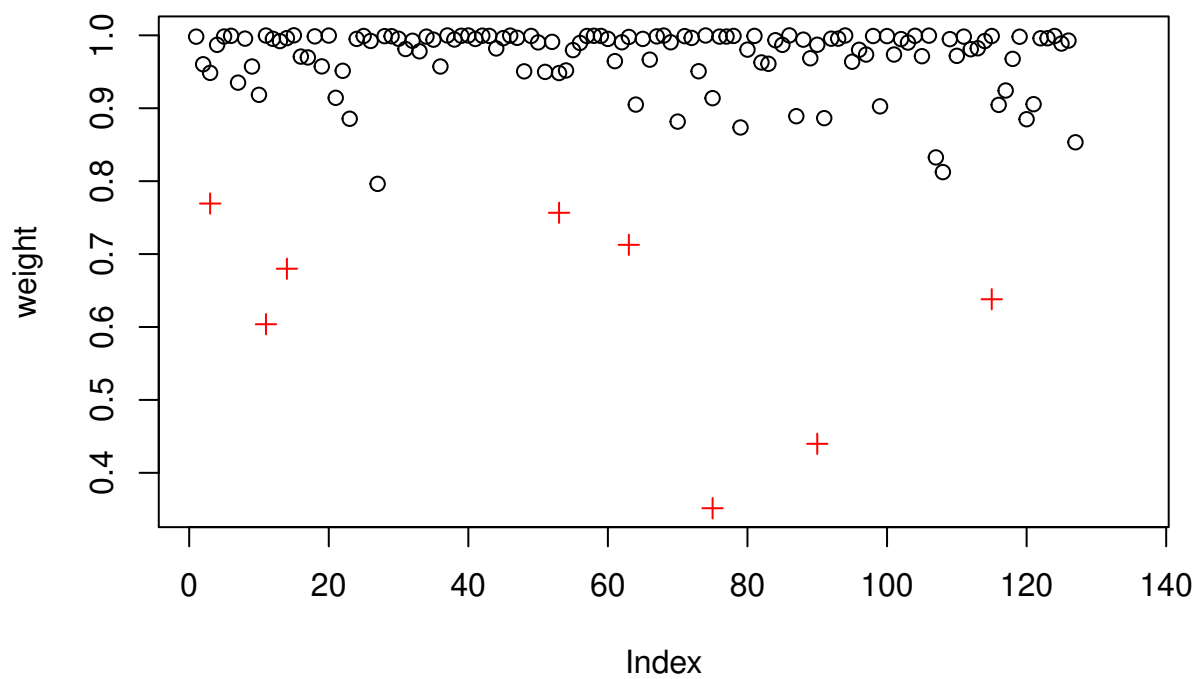
---

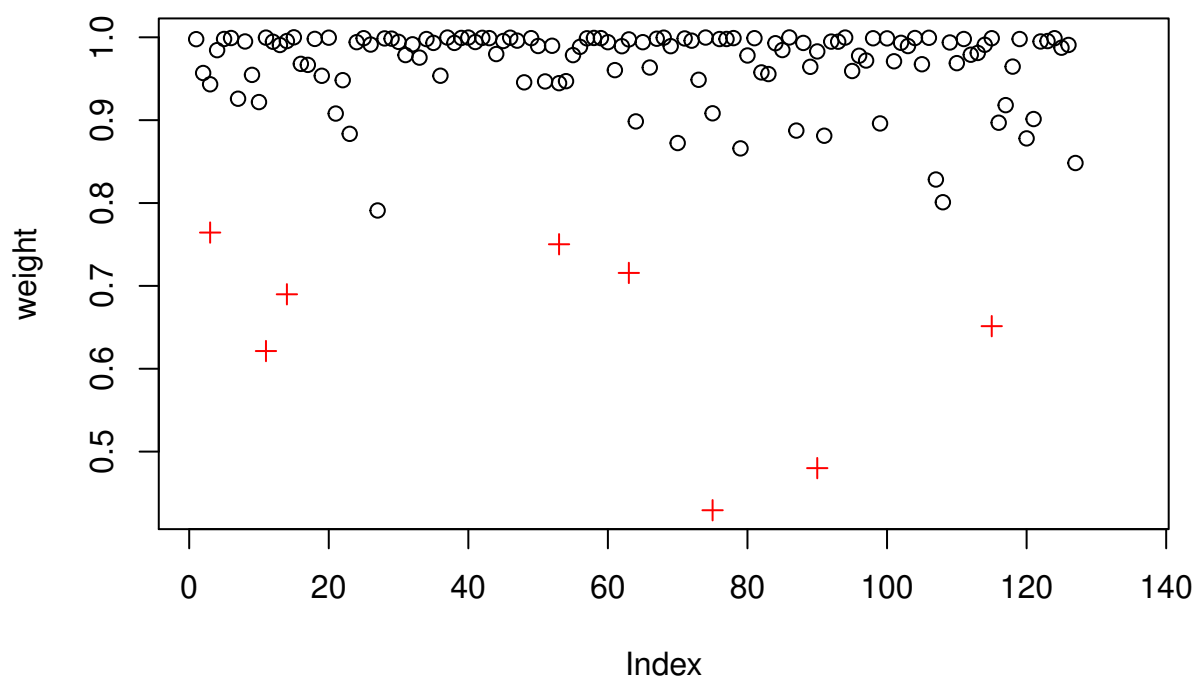
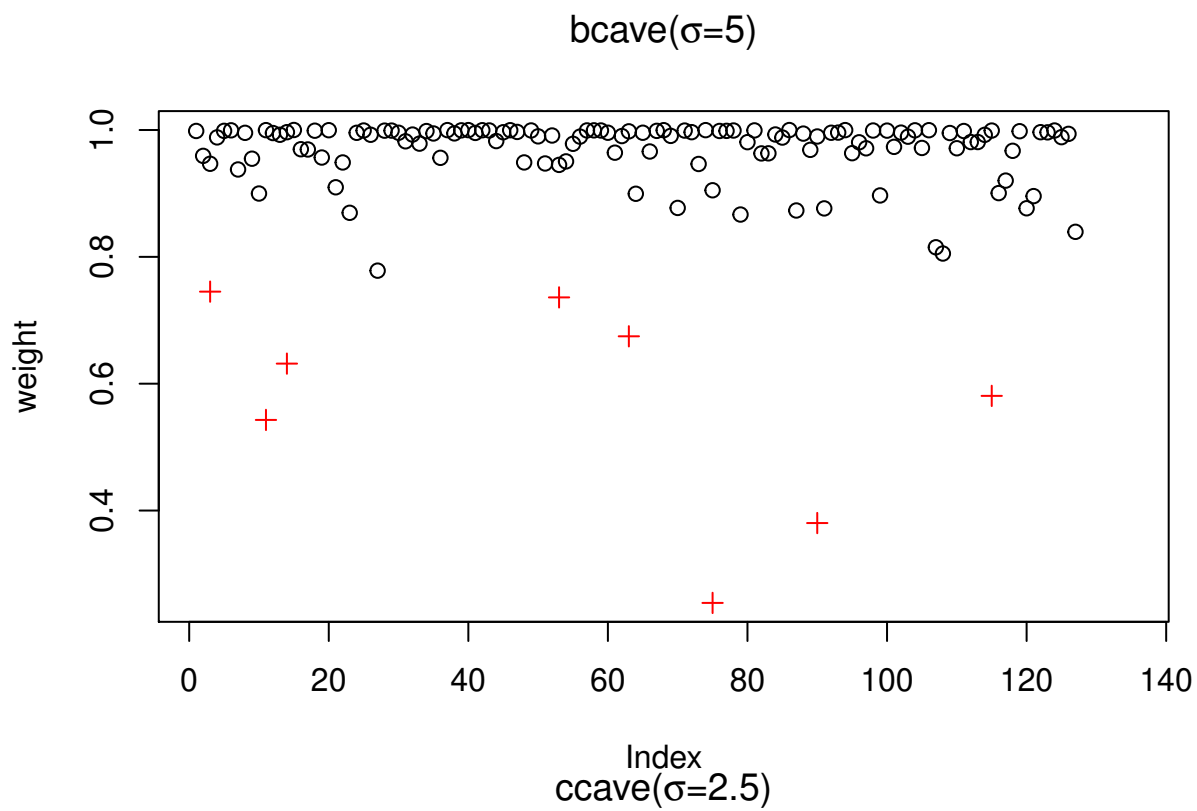
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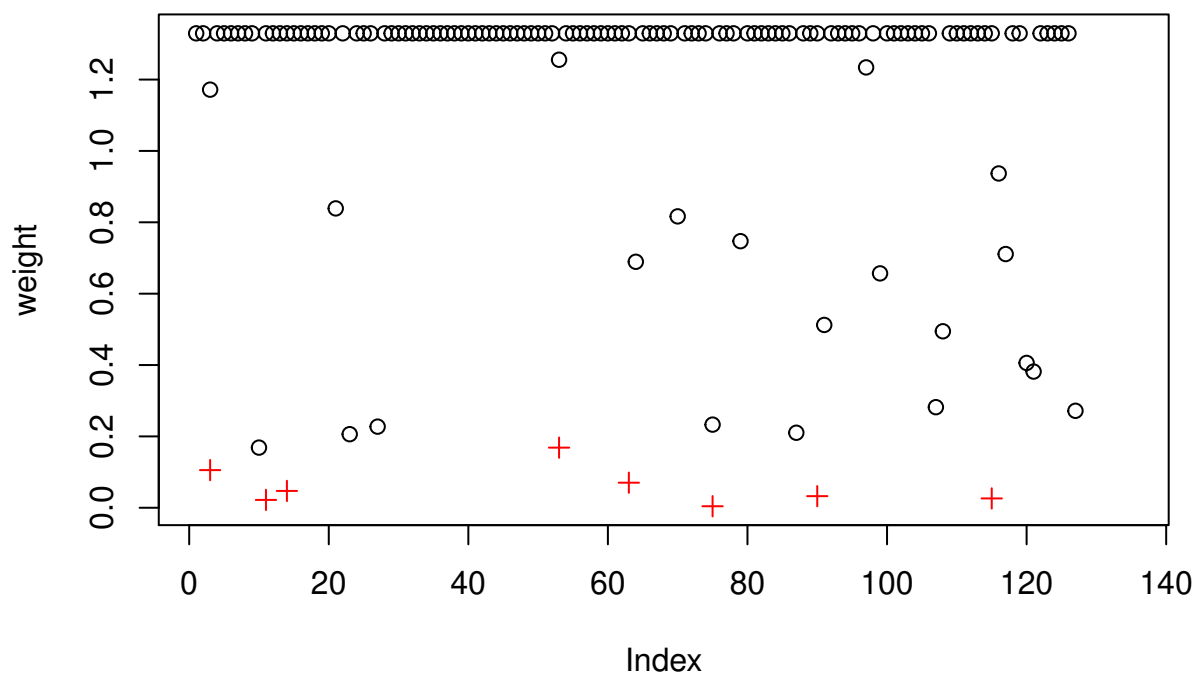
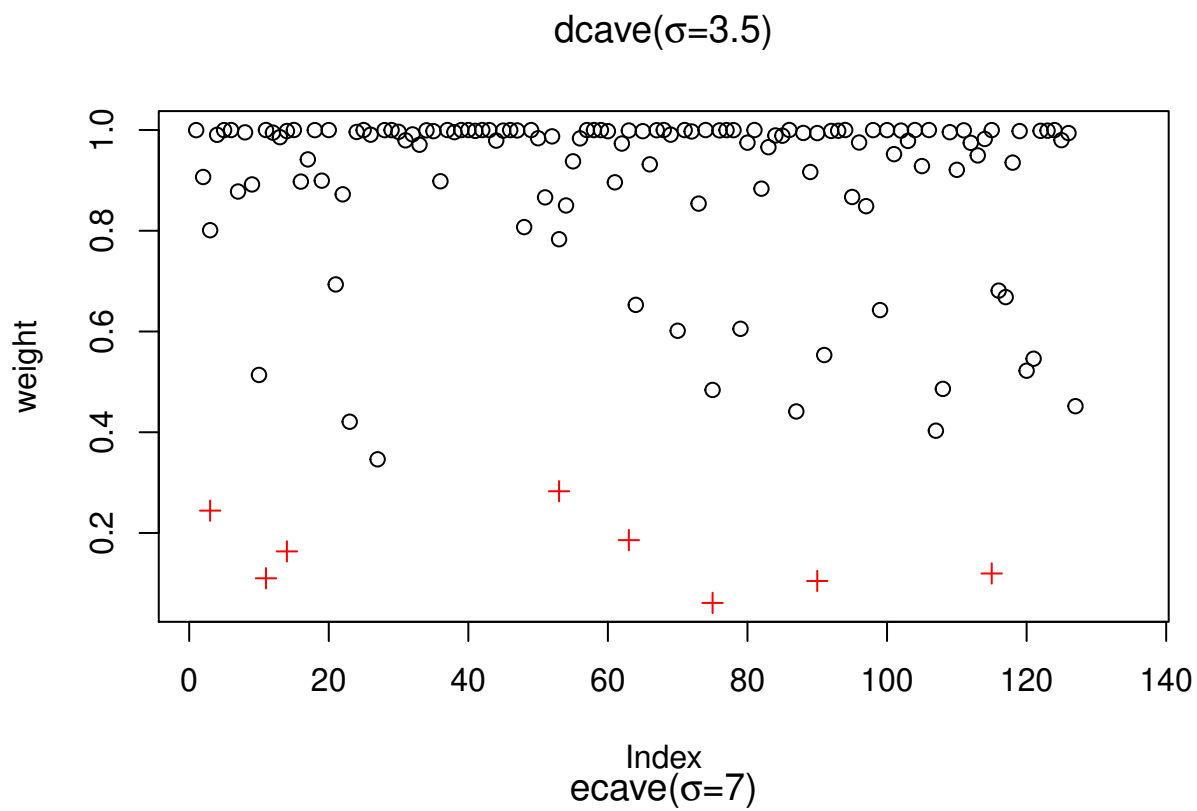
hcave( $\sigma=1.5$ )

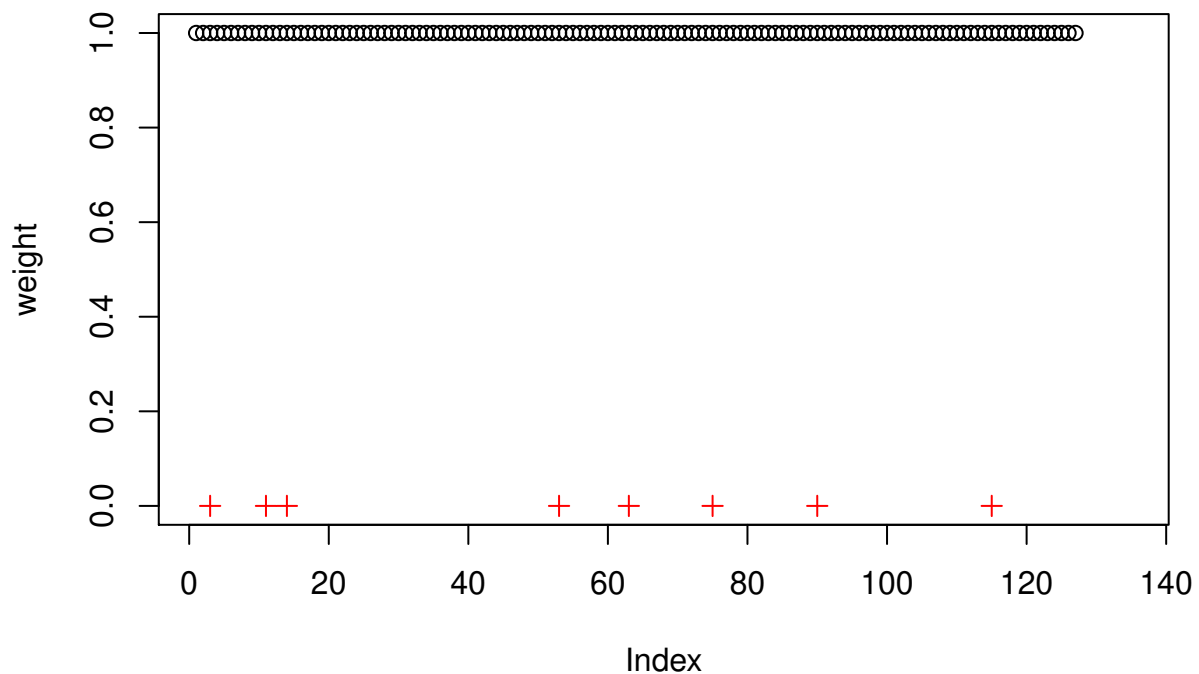
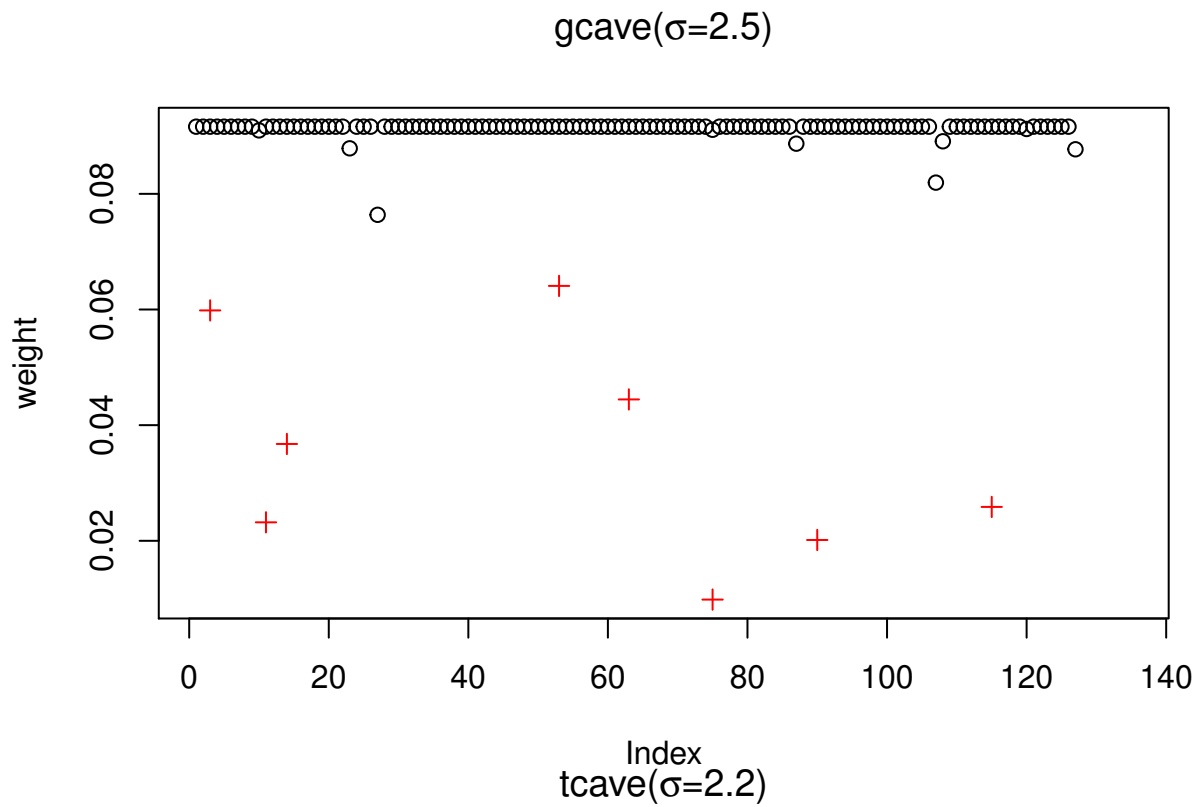


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acave( $\sigma=1.5$ )









Despite large estimated probability  $\geq 0.8$  of trying to breastfeed or not in a logistic regression, these individuals took the opposite decisions.

```
fit.glm <- glm(as.integer(breast)-1~., data=breastfeed, family=binomial())
id <- c(3, 11, 14, 53,63, 75,90, 115)
pred <- predict(fit.glm, type="response") ### predicted probabilities
```

```
cbind(breastfeed, pred)[id,]
```

```
##      breast pregnancy howfed howfedfr partner smokenow smokebf age educat
## 3   Bottle Beginning Breast   Breast Partner      No      No  39    16
## 12  Bottle Beginning Breast   Breast Single      No      No  29    18
## 15  Bottle Beginning Bottle   Breast Partner      No      No  33    21
## 56  Bottle           End Bottle   Breast Partner      No      No  25    16
## 66  Breast           End Bottle   Bottle Partner     Yes     Yes  27    16
## 78  Bottle Beginning Breast   Bottle Partner      No      No  28    28
## 93  Breast Beginning Bottle   Bottle Single      Yes     Yes  19    16
## 118 Bottle Beginning Breast   Breast Single      No      No  20    18
##      ethnic      pred
## 3      White 0.77424035
## 12 Non-white 0.89871707
## 15      White 0.83665383
## 56      White 0.82204044
## 66      White 0.18544970
## 78 Non-white 0.97026037
## 93      White 0.02275785
## 118 Non-white 0.87454933
```

For variable selection, we develop a usual penalized LASSO logistic regression, where the optimal penalty parameter  $\lambda$  is chosen by a 10-fold cross-validation.

```
n.cores <- 2
set.seed(195)
fitcv.glm <- cv.glmreg(as.integer(breast)-1~., data=breastfeed, penalty="enet",
                      family="binomial", type="loss", plot.it=FALSE, parallel=TRUE,
                      n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized logistic regression

```
min(fitcv.glm$cv)
```

```
## [1] -7.815975
```

Penalized logistic regression with penalty LASSO

```
coef(fit)[,fitcv.glm$lambda.which]
```

```
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      -2.492768348      -0.552721156      0.257557827      1.220755264
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.862448995      -2.252414341      0.623626979      0.002488979
##      educat      ethnicNon-white
##      0.119557901      1.541320581
```

Compute the SCAD logistic regression, where the optimal penalty parameter  $\lambda$  is chosen by a 10-fold cross-validation. The SCAD logistic regression is more sparse than the LASSO estimator.

```
set.seed(195)
fitcv.glm <- cv.glmreg(as.integer(breast)-1~., data=breastfeed, penalty="snet",
                      family="binomial", type="loss", plot.it=FALSE, parallel=TRUE,
                      n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized logistic regression

```
min(fitcv.glm$cv)
```

```
## [1] -7.815975
```

Penalized logistic regression with penalty SCAD

```
coef(fit)[,fitcv.glm$lambda.which]
```

```
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      0.09874844      0.00000000      0.00000000      1.04702265
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.47532632      -2.00125709      0.00000000      0.00000000
##      educat      ethnicNon-white
##      0.00000000      1.94414156
```

The  $\lambda$  value in SCAD is then utilized to compute binomial-induced SCAD CC-estimators for various concave components.

```
for(i in c(1:5,8,6,7)){
  cat("\ncfun=", cfun[i], "\n")
  fit.ccglmreg <- ccglmreg(breast~., data=breastfeed, s=sval[i], cfun=i, penalty="snet",
    lambda=fitcv.glm$lambda.optim, dfun=binomial(), parallel=FALSE,
    type.path="nonactive", standardize=TRUE)
  print(coef(fit.ccglmreg))
}
```

```
##
## cfun= hcave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      -0.20262257      0.00000000      0.00000000      1.41623162
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.24121875      -2.31220066      0.00000000      0.00000000
##      educat      ethnicNon-white
##      0.02524874      2.48775264
##
```

```
## cfun= acave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      0.323505087      0.00000000      0.00000000      1.194139787
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.197984927      -2.383687907      0.00000000      0.000000000
##      educat      ethnicNon-white
##      0.008521202      2.523000073
##
```

```
## cfun= bcave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      0.32721541      0.00000000      0.00000000      1.20998505
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.12916071      -2.44460015      0.00000000      0.000000000
##      educat      ethnicNon-white
##      0.01285265      2.63969594
##
```

```
## cfun= ccave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      0.354230145      0.00000000      0.00000000      1.177822530
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.224698662      -2.376636566      0.00000000      0.000000000
```

```
##          educat      ethnicNon-white
##      0.006084647      2.483034951
##
## cfun= dcave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      2.71145780      0.00000000      0.12034058      0.02718298
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.00000000      -3.89297984      0.00000000      0.00000000
##          educat      ethnicNon-white
##      0.00000000      1.15836471
##
## cfun= ecave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      3.26756873      0.00000000      0.00000000      0.05330153
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.00000000      -4.24833062      0.00000000      0.00000000
##          educat      ethnicNon-white
##      0.00000000      2.44864324
##
## cfun= gcave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      -0.70232531      0.00000000      0.00000000      1.75532959
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.00000000      -2.68637620      0.00000000      0.00000000
##          educat      ethnicNon-white
##      0.06456509      3.24729928
##
## cfun= tcave
##      (Intercept) pregnancyBeginning      howfedBreast      howfedfrBreast
##      -2.2679907      0.00000000      0.00000000      1.2678561
##      partnerPartner      smokenowYes      smokebfYes      age
##      0.00000000      -2.4827295      0.00000000      0.00000000
##          educat      ethnicNon-white
##      0.1648061      3.5883907
```

## Robust Poisson regression

A cohort of 3066 Americans over the age of 50 were studied on health care utilization, doctor office visits Heritier et al. (2009). The survey also contained 24 predictors in demographic, health needs and economic access. We compute Poisson-induced CC-estimators, i.e., robust Poisson regressions. The seven smallest weights occur to the subjects with 200, 208, 224, 260, 300, 365 and 750 doctor visits in two years.

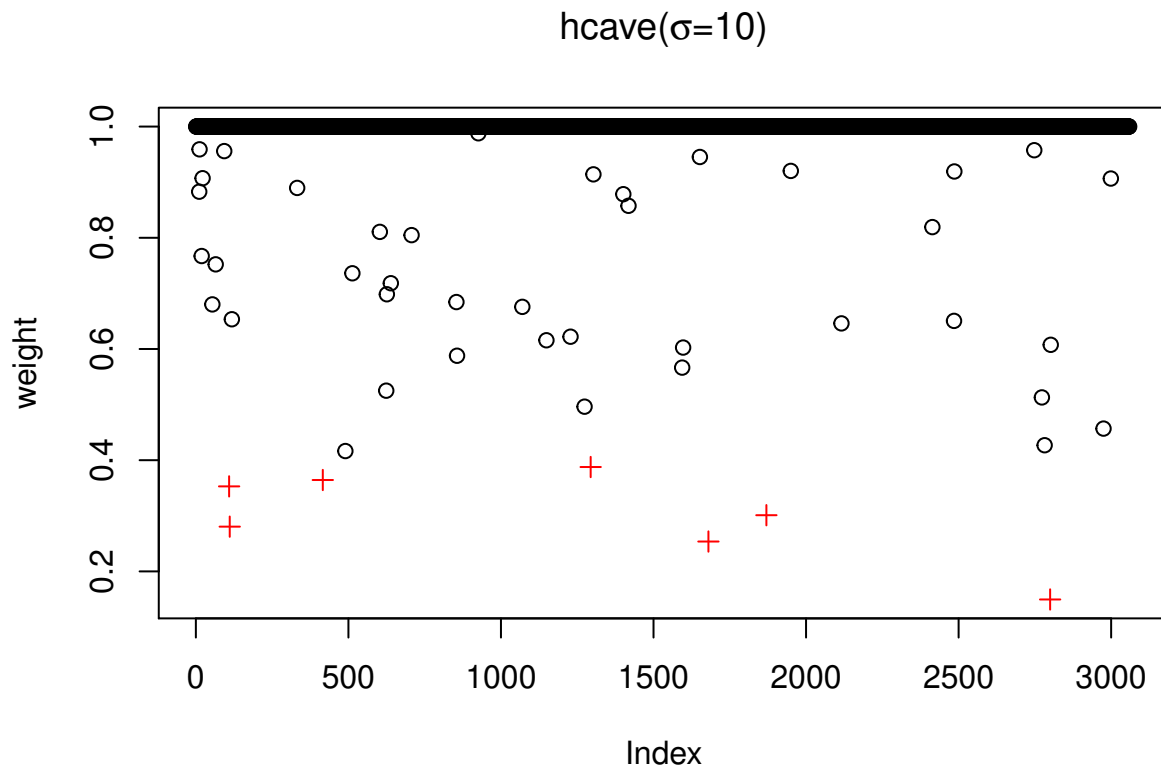
```
data(docvisits)
sval <- c(10, 10, 45, 20, 5, 5, 280, 200)
cfun <- c("hcave", "acave", "bcave", "ccave", "dcave", "gcave", "tcave", "ecave")
id <- 1:7
for(i in c(1:5,8,6,7)){
  fitnew <- ccglm(visits~age+factor(gender)+factor(race)+factor(hispan)
    +factor(marital)+factor(arthri)+factor(cancer)
    +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
    +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
    +edyears+feduc+meduc+log(income+1)+factor(insur),
    data=docvisits,cfun=i,s=sval[i],dfun=poisson(),trace=FALSE)
```

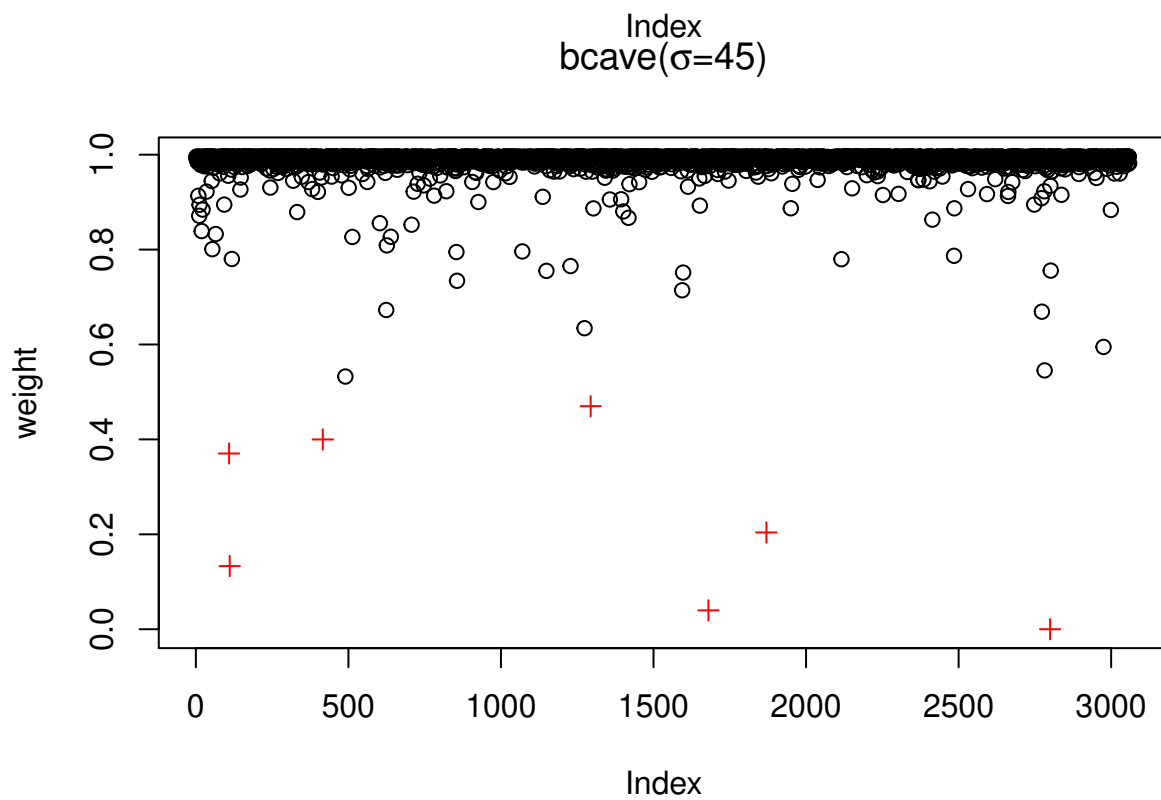
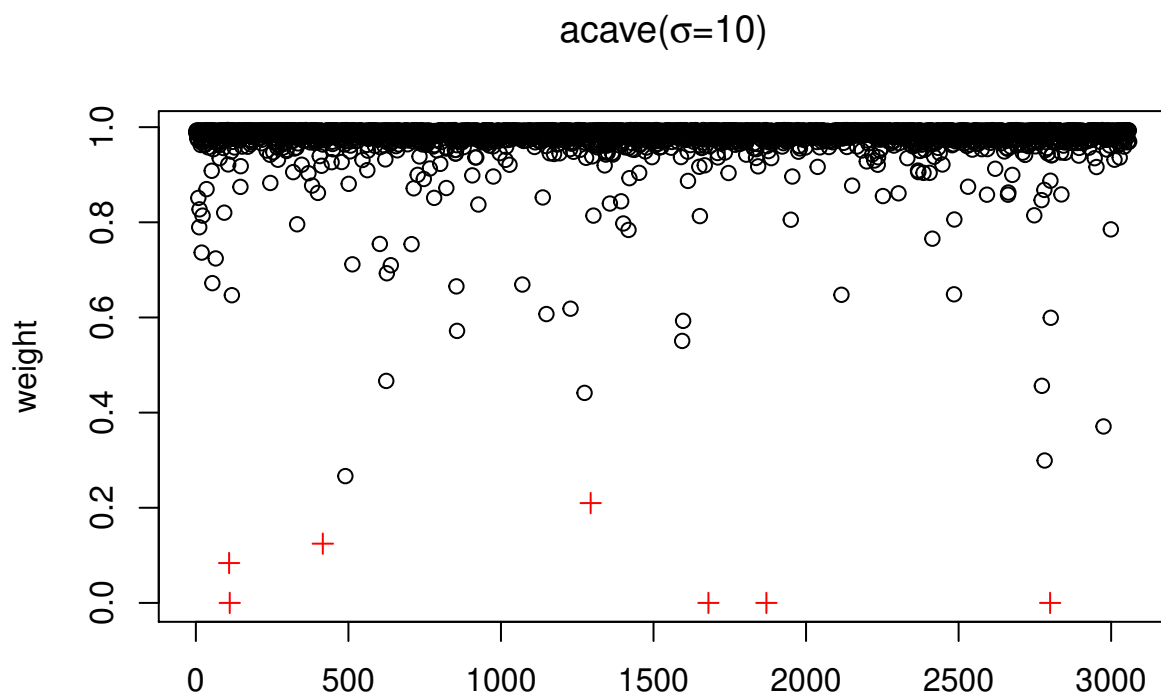


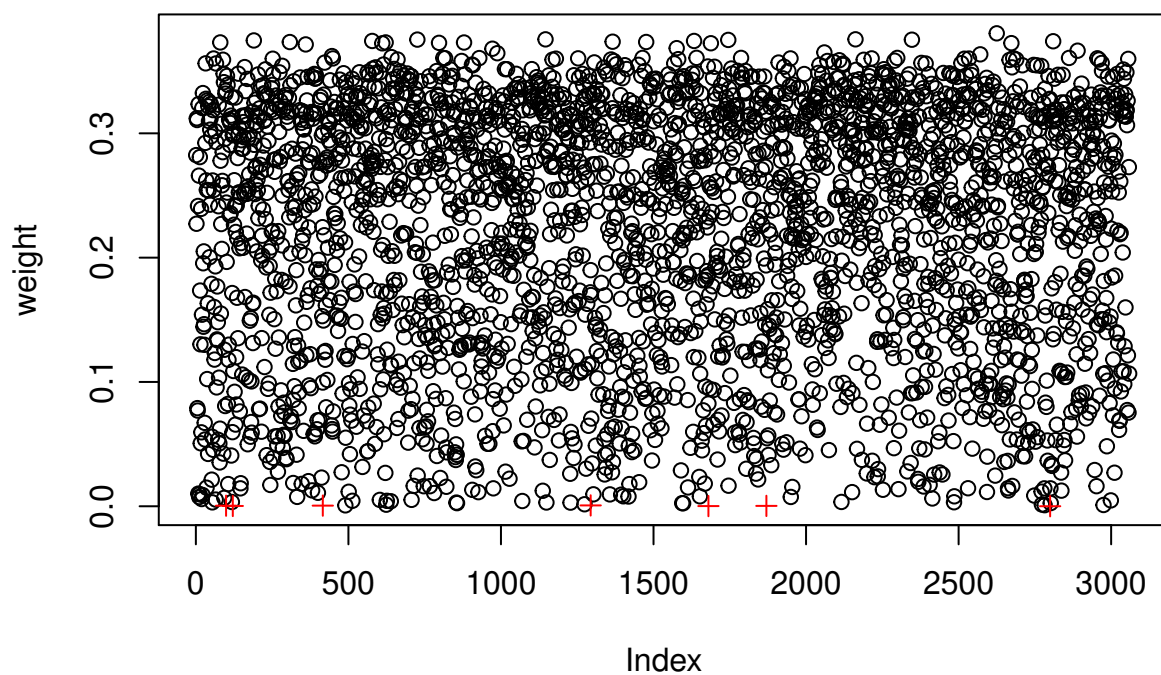
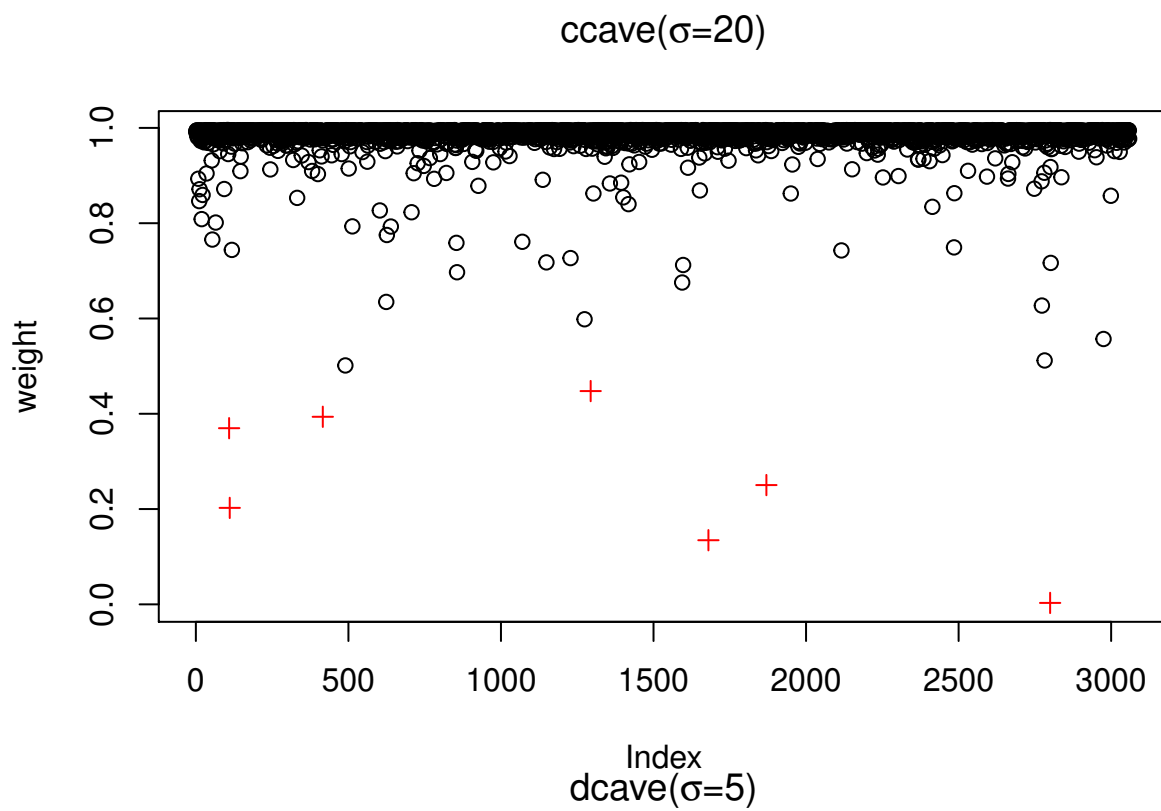
```

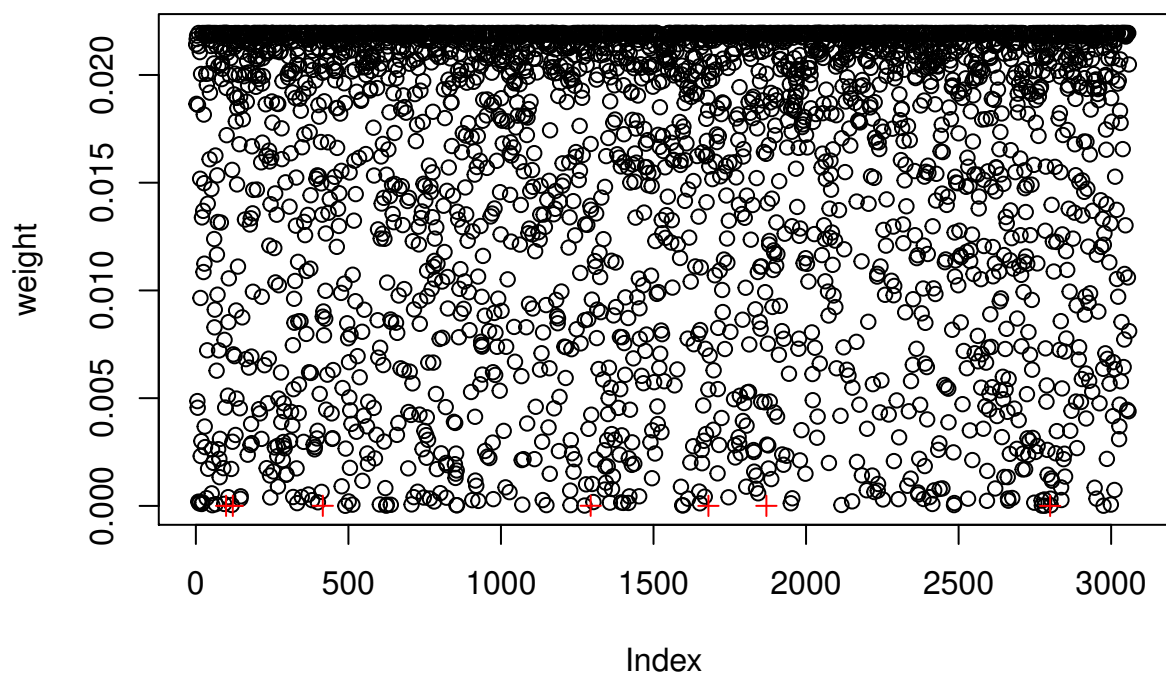
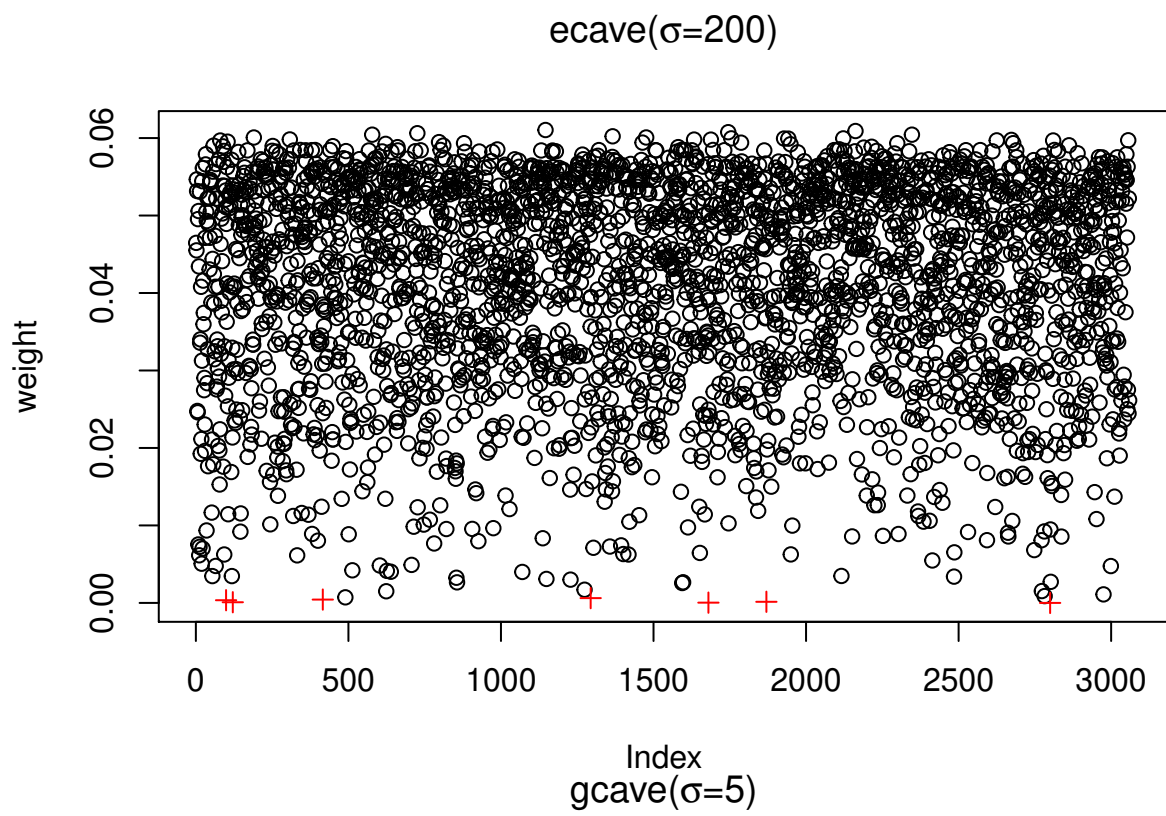
goodid <- sort.list(fitnew$weights_update)[id]
plot(fitnew$weights_update, type="n", ylab="weight",
     main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")"))),
                 list(cfun=cfun[i], s = sval[i]))))
points(fitnew$weights_update[-goodid], ylab="weight",
       main = eval(substitute(expression(paste(cfun, "(", sigma, "=", s, ")"))),
                   list(cfun=cfun[i], s = sval[i]))))
if(i > 4){
  ### deal with overlapped points: obs 109, 111
  x <- sort.list(fitnew$weights_update)[id]
  y <- sort(fitnew$weights_update)[id]
  xnew <- sort(x)
  ynew <- y[sort.list(x)]
  points(xnew[1]-10, ynew[1], pch=3, col="red")
  points(xnew[2]+10, ynew[2], pch=3, col="red")
  points(xnew[3:7], ynew[3:7], pch=3, col="red")
}
else points(sort.list(fitnew$weights_update)[id], sort(fitnew$weights_update)[id],
           pch=3, col="red")
}

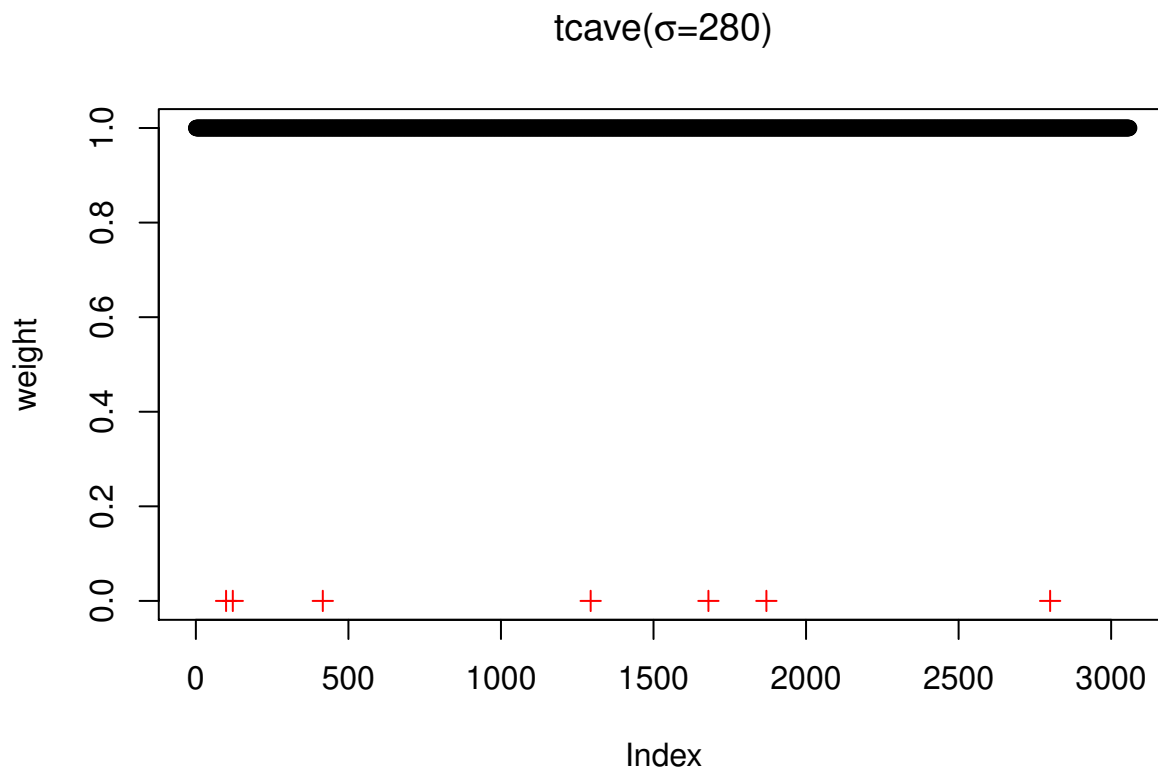
```











Outliers of office visits

```
newid <- sort(sort.list(fitnew$weights_update)[id])
docvisits$visits[newid]
```

```
## [1] 224 300 208 200 365 260 750
```

Penalized Poisson regression with LASSO penalty. The tuning parameter  $\lambda$  value is chosen by cross-validation.

```
set.seed(195)
fitcv.glm <- cv.glmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
  +factor(marital)+factor(arthri)+factor(cancer)
  +factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
  +factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
  +edyears+feduc+meduc+log(income+1)+factor(insur),
  data=docvisits,family="poisson", penalty="enet", type="loss",
  plot.it=FALSE, parallel=TRUE, n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized Poisson regression

```
min(fitcv.glm$cv)
```

```
## [1] -2894.529
```

Penalized Poisson regression with penalty LASSO

```
coef(fit)[,fitcv.glm$lambda.which]
```

```
##      (Intercept)          age factor(gender)1  factor(race)1
##      1.958525362    -0.006577549    0.058079225    -0.016603770
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.005810267    0.000000000    0.087497840    0.115530929
## factor(hipress)1 factor(diabet)1    factor(lung)1 factor(hearth)1
```

```
##      0.139162108      0.272014643      0.071727840      0.264918089
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.069391756      0.220222838      0.068125763      0.043447857
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.068932457      0.312374326      0.616403088      0.545308893
##      edyears      feduc      meduc      log(income + 1)
##      0.009212421      -0.023840699      0.000000000      0.058544656
## factor(insur)1
##      0.072032891
```

Penalized Poisson regression with SCAD penalty. The tuning parameter  $\lambda$  value is chosen by cross-validation.

```
set.seed(195)
fitcv.glm <- cv.glmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
+factor(marital)+factor(arthri)+factor(cancer)
+factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
+factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
+edyears+feduc+meduc+log(income+1)+factor(insur),
data=docvisits, family="poisson", penalty="snet", type="loss",
plot.it=FALSE, parallel=TRUE, n.cores=n.cores, standardize=TRUE)
fit <- fitcv.glm$fit
```

The smallest CV value from penalized Poisson regression

```
min(fitcv.glm$cv)
```

```
## [1] -2894.529
```

Penalized Poisson regression with penalty SCAD

```
coef(fit)[,fitcv.glm$lambda.which]

##      (Intercept)      age factor(gender)1 factor(race)1
##      1.858069526      -0.003786934      0.000000000      0.000000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000000      0.000000000      0.029059308      0.067717808
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.120234297      0.296950385      0.000000000      0.291999839
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.001233415      0.252816298      0.000000000      0.000000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.000000000      0.368482223      0.681643963      0.641110056
##      edyears      feduc      meduc      log(income + 1)
##      0.004754850      0.000000000      0.000000000      0.040644933
## factor(insur)1
##      0.017165257
```

The  $\lambda$  value in SCAD is then utilized to compute robust Poisson SCAD CC-estimators for various concave components.

```
for(i in c(1:5,8,6,7)){
  cat("\nccfun=", cfun[i], "\n")
  fit.ccglmreg <- ccglmreg(visits~age+factor(gender)+factor(race)+factor(hispan)
+factor(marital)+factor(arthri)+factor(cancer)
+factor(hipress)+factor(diabet)+factor(lung)+factor(hearth)
+factor(stroke)+factor(psych)+factor(iadla)+factor(adlwa)
+edyears+feduc+meduc+log(income+1)+factor(insur),
data=docvisits, s=sval[i], cfun=i, penalty="snet",
```

```

lambda=fitcv.glm$lambda.optim, dfun=poisson(), parallel=FALSE,
type.path="nonactive", standardize=TRUE)
print(coef(fit.ccglmreg))
}

```

```

##
## cfun= hcave
##      (Intercept)          age factor(gender)1 factor(race)1
##      1.98635667      0.00000000      0.00000000      0.00000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.00000000      0.00000000      0.03808422      0.03052763
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.10857169      0.22472945      0.01353496      0.32244446
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.04586816      0.26510976      0.00000000      0.00000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.00000000      0.25088902      0.43649856      0.53724280
##      edyears          feduc          meduc log(income + 1)
##      0.00000000      0.00000000      0.00000000      0.00000000
## factor(insur)1
##      0.00000000
##
## cfun= acave
##      (Intercept)          age factor(gender)1 factor(race)1
##      1.98189657      0.00000000      0.00000000      0.00000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.00000000      0.00000000      0.05203613      0.03289539
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.08280116      0.19573875      0.02543506      0.33252481
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.06802881      0.28272718      0.00000000      0.00000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.00000000      0.14459462      0.36550972      0.48845297
##      edyears          feduc          meduc log(income + 1)
##      0.00000000      0.00000000      0.00000000      0.00000000
## factor(insur)1
##      0.00000000
##
## cfun= bcave
##      (Intercept)          age factor(gender)1 factor(race)1
##      1.978829e+00 -5.216532e-05      0.000000e+00      0.000000e+00
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000e+00      0.000000e+00      3.802145e-02      1.872132e-02
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      1.222130e-01      1.954667e-01      2.796854e-02      3.279856e-01
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      7.044509e-02      2.903266e-01      0.000000e+00      0.000000e+00
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.000000e+00      2.729839e-01      3.864905e-01      5.060047e-01
##      edyears          feduc          meduc log(income + 1)
##      0.000000e+00      0.000000e+00      0.000000e+00      0.000000e+00
## factor(insur)1
##      0.000000e+00
##

```

```

## cfun= ccave
##      (Intercept)                age factor(gender)1 factor(race)1
##      1.976567e+00 -4.375527e-05  0.000000e+00  0.000000e+00
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000e+00  0.000000e+00  3.463168e-02  1.715012e-02
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      1.290332e-01  1.881076e-01  2.466065e-02  3.285646e-01
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      6.225529e-02  2.828722e-01  0.000000e+00  0.000000e+00
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.000000e+00  2.736037e-01  3.977317e-01  5.175466e-01
##      edyears          feduc          meduc log(income + 1)
##      0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
## factor(insur)1
##      0.000000e+00
##
## cfun= dcave
##      (Intercept)                age factor(gender)1 factor(race)1
##      1.82551207  0.00000000  0.00000000  0.00000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.00000000  0.00000000  0.02685254  0.00000000
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.05332835  0.02872780  0.00000000  0.35662745
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.00000000  0.03407841  0.00000000  0.00000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.00000000  0.00000000  0.00000000  0.60245837
##      edyears          feduc          meduc log(income + 1)
##      0.00000000  0.00000000  0.00000000  0.00000000
## factor(insur)1
##      0.00000000
##
## cfun= ecave
##      (Intercept)                age factor(gender)1 factor(race)1
##      1.881486009  0.000000000  0.000000000  0.000000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000000  0.000000000  0.034132803  0.006426225
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.066216272  0.067616141  0.000000000  0.346514697
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.000000000  0.080270550  0.000000000  0.000000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.000000000  0.046662355  0.359270807  0.592414299
##      edyears          feduc          meduc log(income + 1)
##      0.000000000  0.000000000  0.000000000  0.000000000
## factor(insur)1
##      0.000000000
##
## cfun= gcave
##      (Intercept)                age factor(gender)1 factor(race)1
##      1.78471352  0.000000000  0.000000000  0.000000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000000  0.000000000  0.02635143  0.000000000
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1

```



```

##      0.06526733      0.01393760      0.00000000      0.34003347
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.00000000      0.02149100      0.00000000      0.00000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.00000000      0.00000000      0.00000000      0.65357731
##      edyears      feduc      meduc log(income + 1)
##      0.00000000      0.00000000      0.00000000      0.00000000
## factor(insur)1
##      0.00000000
##
## cfun= tcave
##      (Intercept)      age factor(gender)1 factor(race)1
##      1.969315265      0.000000000      0.000000000      0.000000000
## factor(hispan)1 factor(marital)1 factor(arthri)1 factor(cancer)1
##      0.000000000      0.000000000      0.064288784      0.030497991
## factor(hipress)1 factor(diabet)1 factor(lung)1 factor(hearth)1
##      0.076827493      0.244339304      0.030909720      0.325954228
## factor(stroke)1 factor(psych)1 factor(iadla)1 factor(iadla)2
##      0.130262217      0.306230170      0.001956634      0.000000000
## factor(iadla)3 factor(adlwa)1 factor(adlwa)2 factor(adlwa)3
##      0.000000000      0.200614987      0.372065923      0.460492140
##      edyears      feduc      meduc log(income + 1)
##      0.001441250      0.000000000      0.000000000      0.000000000
## factor(insur)1
##      0.000000000

```

## References

- Heritier, Stephane, Eva Cantoni, Samuel Copt, and Maria-Pia Victoria-Feser. 2009. *Robust Methods in Biostatistics*. Vol. 825. John Wiley & Sons.
- Wang, Zhu. 2020. “Unified Robust Estimation via the COCO.” *arXiv E-Prints*, October, arXiv:2010.02848. <http://arxiv.org/abs/2010.02848>.