

# Getting Started with Mokken Scale Analysis in R

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# Chapter 1

## Getting started

### 1.1 Introduction

This report aims at researchers who have Windows installed on their computer and who wish to conduct Mokken scale analysis using the freeware R-package `mokken` (Van der Ark, 2007) but who do not know anything about R. It is a step by step guide from scratch to actually performing Mokken scale analysis. A more elaborate book on learning R for SPSS and SAS users is Muenchen (2008). The report is organized as follows. In this chapter (chap. 1), I discuss the preparations that are needed before `mokken` can be used. In section 1.2, I discuss the installation of R and all the necessary packages. In section 1.3, I discuss how SPSS, SAS, STATA, and Splus data sets should be converted to R. In section 1.4, I show a few R commands that come in handy for data manipulation (e.g., variable selection). In chapter 2, I explain `mokken`. In section 2.1, I give an overview of `mokken`'s most important commands (known as *functions* in R) illustrated by examples. In section 3, I explain the use of `mokken` by showing the code for most analyses in the book *Introduction to nonparametric item response theory* (Sijtsma & Molenaar, 2003).

### 1.2 Installation

#### 1.2.1 What is R?

R (R Development Core Team, 2006) is a language and environment for statistical computing and graphics. It is something in between a statistical package such as SPSS, STATA, or SAS and a programming language such as C++, PASCAL, or FORTRAN. It has two big advantages: It is for free and it has open source. Because it is for free it is accessible for anyone at any time, and because it has an open source researchers can add *packages* to R. Currently, over 2100 packages are added — the package `mokken` is one of

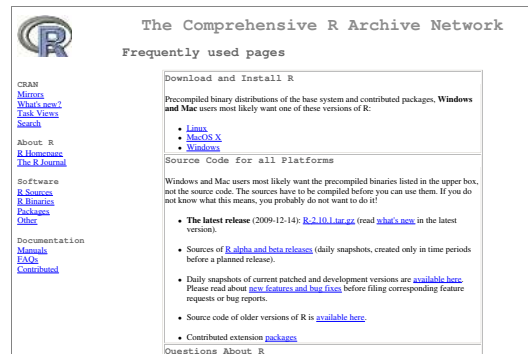


Figure 1.1: *R* website. Click on [Windows](#).

them — allowing the user to conduct almost every possible statistical procedure ranging from common statistical procedures, which are also available in commercial software packages, to statistical techniques such as item response theory, spectral analysis, marginal modelling, Bayesian analysis, and latent class analysis. Every two months or so R releases a new version. At the time of writing version R-2.10.1 was the most recent version. Although, it is good to have a recent version of R, I only update a new version once a year or so. The major R website is <http://cran.r-project.org/>. It contains an abundance of information. A special page is devoted to packages that are of interest to psychometricians (<http://cran.r-project.org/web/views/Psychometrics.html>).

## 1.2.2 Installing R

Installing R requires the following steps

1. Go to <http://cran.r-project.org/>.
2. Click on [Windows](#) (See Figure 1.1)
3. Click on [base](#) (See Figure 1.2)
4. Click on [Download R 2.10.1 for Windows](#) (or a more recent version; see Figure 1.3)
5. Save the file `R-2.10.1-win32.exe` on your computer (e.g., on `C:/`).
6. Run the file `R-2.10.1-win32.exe` from your computer. You can choose all the default values in the installation Wizard.

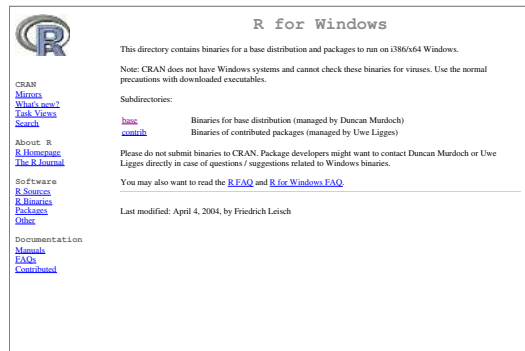


Figure 1.2: *R website. Click on [base](#).*

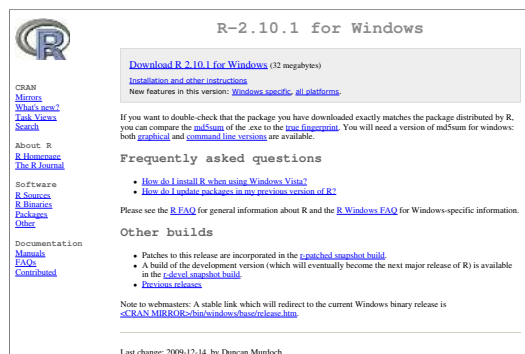


Figure 1.3: *R website. Click on [Download R 2.10.1 for Windows](#).*

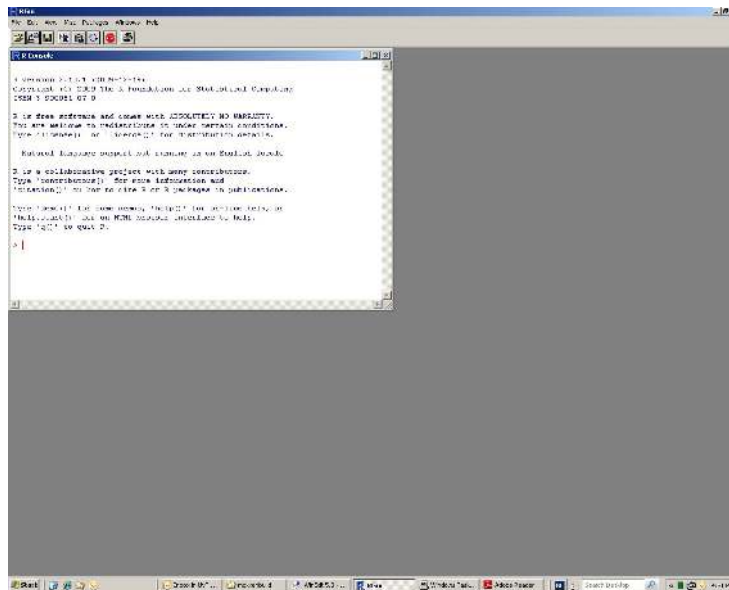


Figure 1.4: *R console*.

7. R will be available from the desk top icon, and from the programme's menu.

### 1.2.3 Installing the package mokken

1. Open R (Figure 1.4 shows the *R console*).
2. In the pull down menu choose **Packages**, **Install package(s)**, choose a location nearby you, and choose the package **mokken** (Figure 1.5).

The package **mokken** is now installed on your computer and need not be installed anymore. It may be noted that the same procedure applies to the installation of all packages.

### 1.2.4 Working with R

Except for loading packages (section 1.2.3), almost everything in R is conducted by typing (or pasting) code in the R console (Figure 1.4) just after the prompt (**>**) and end with hard return (Enter). Code to be typed is printed in *slanted typewriter text* and preceded by a **>** (a prompt). The resulting output is printed exactly as it is printed on the screen in **typewrite text**. Note that R is case sensitive. Some examples.

```
> 6 - 3
```

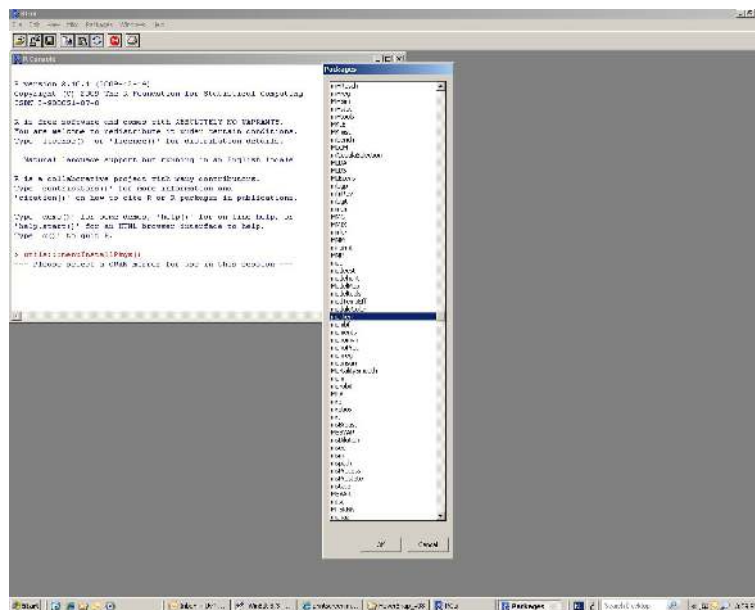


Figure 1.5: *R console*. Choose mokken.

```
[1] 3
```

```
> options(width = 60)
> x <- sqrt(2)
> x
```

```
[1] 1.414214
```

R assigns the value  $\sqrt{2}$  to variable  $x$  (line 1) and displays the value of  $x$  (line 2)

To load the procedures of **mokken** into the memory of R type

```
> library(mokken)
```

To quit R, type

```
> q()
```

Free of charge introductions to R are available on the Internet.

- R Development Core Team (2009). *An Introduction to R*. Retrieved from <http://cran.r-project.org/doc/manuals/R-intro.html> (html) or <http://cran.r-project.org/doc/manuals/R-intro.pdf> (pdf).

- Paradis, E. (2005). *R for beginners*. Retrieved from [http://cran.r-project.org/doc/contrib/Paradis-rdebuts\\_en.pdf](http://cran.r-project.org/doc/contrib/Paradis-rdebuts_en.pdf)
- Baron, J., & Li, Y. (2007). *Notes on the use of R for psychology experiments and questionnaires*. Retrieved from <http://www.psych.upenn.edu/~baron/rpsych/rpsych.html>

Many more sources are available from <http://cran.r-project.org/> (Contributed Documentation).

## 1.3 Converting data to R and back again

Converting a data set from a commercial package to R is the Achilles Heel of Mokken scale analysis in R. Commercial packages have no interest in free software that can easily read their data sets and these companies put no effort making their data files compatible with R. As a result, small things that you may not be aware of (e.g., whether your computer uses a point or a comma as a decimal separator, whether or not the rows in your data set have labels) may affect the conversion. An elaborated manual for converting many types of files in to files that can be read by R is available from <http://cran.r-project.org/doc/manuals/R-data.html>. Here only conversions to and from SPSS, SAS, STATA, and Splus are briefly discussed. The fastest strategy is to read the SPSS, SAS, STATA, or Splus file directly in R. Direct reading may occasionally go wrong and an alternative option is to save the SPSS, SAS, STATA, or Splus file as a text-only file (ASCII file), and read the ASCII file into R. In the latter procedure, the variable names may get lost.

### 1.3.1 SPSS files

#### Converting SPSS files directly

I assume that an SPSS data set named `ExampleSPSS.sav` has been saved on `C:/`<sup>1</sup>.

1. Type the following code in the R console

```
> library(foreign)
> ExampleR <- data.frame(read.spss("C:/ExampleSPSS.sav"))
> fix(ExampleR)
```

---

<sup>1</sup>`ExampleSPSS.sav` is a completely arbitrary name and your data set probably has a different name and may be located on another drive than `C:/`. Therefore, you should replace `C:/ExampleSPSS.sav` by your complete path and file name. The SPSS-file is not included in the Mokken package.

Note that `data.frame()` is an R function; it saves the data in a matrix-like manner, allowing different measurement levels for the scores in each column. Most data sets in R belong to the class `data.frame`. The data file is now stored in the memory of R under the name `ExampleR`<sup>2</sup>. The last command is not necessary. It opens the R data in a spreadsheet in another window in R; the spreadsheet can be used to check whether the transformation went well. If necessary, the spread sheet may be modified. If the spreadsheet window is closed (by clicking the close button in the upper right-hand corner, see Figure 1.6) the changes are saved. Note that `library(foreign)` may be omitted, if it has been typed in before during the same R session.

2. If R is closed, `ExampleR` are lost. Therefore, the data should be saved in an R format that can be retrieved easily. To save the data (in the file `C:/ExampleR.Rdata`) type

```
> save(ExampleR, file = "C:/ExampleR.Rdata")
```

To get the data back into R type

```
> load("C:/ExampleR.Rdata")
```

### **Saving SPSS files as ASCII files and read the ASCII files**

Save the data as a tab delimited ASCII file (.dat file) This format can be read easily by R. The SPSS syntax is

```
SAVE TRANSLATE OUTFILE='C:\ExampleSPSS2.dat'
/TYPE=TAB
/MAP
/REPLACE
/FIELDNAMES
/CELLS=LABELS.
```

### **Converting R data to SPSS**

To convert R data sets to SPSS directly is impossible. R creates an SPSS syntax file and an ASCII data file. The SPSS syntax file should be run within SPSS. To create the syntax file `ExampleSPSS.SPS` and the data file `ExampleSSPS.txt` from the R data `ExampleR`, type

```
> library(foreign)
> write.foreign(ExampleR, datafile = "C:/ExampleSPSS.txt",
+   codefile = "C:/ExampleSPSS.SPS", package = "SPSS")
```

---

<sup>2</sup>Again `ExampleR` is a completely arbitrary name and you may decide to name it differently, for example, `NKSPdata2008`.





```
> save(ExampleR, file = "C:/ExampleR.Rdata")
```

To get the data back into R type

```
> load("C:/ExampleR.Rdata")
```

### Converting R data to SAS

To convert R data sets to SAS directly is impossible. R creates a SAS syntax file and an ASCII data file. The SAS syntax file should be run within SAS. To create the syntax file "ExampleSAS.XXX" and the data file "ExampleSAS.txt" from the R data ExampleR, type

```
> library(foreign)
> write.foreign(ExampleR, datafile = "C:/ExampleSAS.txt",
+   codefile = "C:/ExampleSAS.XXX", package = "SAS")
```

### 1.3.3 STATA files

#### Converting STATA files directly

I assume that the STATA data set ExampleSTATA.dta has been saved on C:/.

1. Type the following code in the R console

```
> library(foreign)
> ExampleR <- data.frame(read.dta("C:/ExampleSTATA.dta"))
> fix(ExampleR)
```

The data file is now stored in the memory of R under the name ExampleR. The last command is not necessary. It opens the R data in a spread sheet, which can be used to check whether the transformation went well. If necessary, the spread sheet may be modified. If the spread-sheet window is closed the changes are saved. Note that library(foreign) may be omitted, if it has been typed in before during the same R session.

2. If R is closed, ExampleR are lost. Therefore, the data should be saved in an R format that can be retrieved easily. To save the data (in the file C:/ExampleR.Rdata) type

```
> save(ExampleR, file = "C:/ExampleR.Rdata")
```

To get the data back into R type

```
> load("C:/ExampleR.Rdata")
```

## Converting R data to STATA

To convert R data sets to STATA directly is impossible. R creates a STATA syntax file and an ASCII data file. The STATA syntax file should be run within STATA. To create the syntax file "ExampleSTATA.do" and the data file "ExampleSTATA.dat" from the R data `ExampleR`, type

```
> library(foreign)
> write.foreign(ExampleR, datafile = "C:/ExampleSTATA.dat",
+   codefile = "C:/ExampleSTATA.do", package = "Stata")
```

### 1.3.4 Splus files

#### Converting Splus files directly

I assume that the Splus data set `ExamplePlus.ssc` has been saved on `C:/`.

1. Type the following code in the R console

```
> library(foreign)
> ExampleR <- data.frame(read.s("C:/ExamplePlus.ssc"))
> fix(ExampleR)
```

The data file is now stored in the memory of R under the name `ExampleR`. The last command is not necessary. It opens the R data in a spread sheet, which can be used to check whether the transformation went well. If necessary, the spread sheet may be modified. If the spread-sheet window is closed the changes are saved. Note that `library(foreign)` may be omitted, if it has been typed in before during the same R session.

2. If R is closed, `ExampleR` are lost. Therefore, the data should be saved in an R format that can be retrieved easily. To save the data (in the file `C:/ExampleR.Rdata`) type

```
> save(ExampleR, file = "C:/ExampleR.Rdata")
```

To get the data back into R type

```
> load("C:/ExampleR.Rdata")
```

#### Converting Splus objects to R objects

I assume that the you have an Splus object `ExamplePlus` in Splus, and that all data can be stored in `C:/`. Type in the Splus console

```
> dump(ExamplePlus, "C:/Example.dmp")
```

Next, type in the R console

```
> ExampleR <- dget("C:/Example.dmp")
```

## 1.4 R commands required for mokken

Rather than typing commands in the R console, I advice to type the commands in a plain text file, save the file, and paste a command or a series of commands into R. In this way the commands will not be lost.

- If **mokken** is used, then one should start each R session with

```
> library(mokken)
```

- If help is required at any stage use the command **help()**. For example,

```
> help(mokken)
```

The help file contains examples of **mokken**. It can be instructive to paste these examples into the R console.

- A hash (#) indicates that everything beyond it on the same line is a comment.

does not do anything.

- There are three data sets included in **mokken**: **acl**, **cavalini**, and **transreas**. Typing

```
> data(acl)
> data(cavalini)
> data(transreas)
```

makes them available in R. Note that without these **data()** commands, the data sets are are not available.

```
> help(acl)
```

will give all the information on **acl**

```
> fix(cavalini)
```

will show **cavalini** in a spreadsheet.

- An arrow **<-** is used for assignment. Examples

```
> X <- ac1
> Y <- 3
> Z <- c(1, 2, 3, 8:11)
```

The value of **X** is the data matrix **ac1** (**X** and **ac1** are now equivalent).  
 The value of **Y** is 3. The value of **Z** is the vector (1,2,3,8,9,10,11). It  
 can be verified by typing

```
> Y

[1] 3

> Z

[1] 1 2 3 8 9 10 11
```

- To select columns and rows from the data matrix brackets are used.

```
> X1 <- ac1[, 1]
```

X1 are the scores on the first item 'Reliable')

```
> X2 <- ac1[, 11:20]
```

X2 are the scores on items 11 to 20 (i.e., only the scores on the 10 items  
 of the scale 'Achievement')

```
> X3 <- ac1[1:10, ]
```

X3 are the scores of the first 10 respondents items on all items

```
> X4 <- ac1[232, 133]
```

X4 is the score of respondent 232 on item 133

```
> scale.1 <- c(1, 2, 4)
```

```
> X5 <- ac1[c(1:100, 201:300), scale.1]
```

X5 are the scores of respondents 1-100 and 201-300, on items 1, 2, and  
 4

```
> X6 <- ac1[ac1[, 1] == 2, ]
```

X6 are the scores of those respondents who had a score 2 on item 1.

Note that in data matrices **X3** to **X6**, the cases (rows) not selected are thrown away, and case numbers are not available. Case numbers can be made through the following commands. If you want to identify the them, you can create case numbers for **ac1**.

```
> dimnames(ac1)[[1]] <- 1:nrow(ac1)
```

If you repeat the analyses above, you may observe that the case numbers have been preserved.

## Chapter 2

# The R package mokken

### 2.1 An overview of the functions

The package `mokken` consists of the following functions

#### 2.1.1 `aisp`

Function `aisp` performs Mokken's (1971) automated item selection algorithm. In the example, the scores on the first ten items from ACL are used; these are the items of the scale Communality. Mokken's automated item selection algorithm is applied to the ten items. The output (in blue) shows that items `unscrupulous*` and `unintelligent*` are unscalable, that items `reliable`, `honest`, `deceitful*`, and `dependable` are in scale 1, and items `obnoxious*`, `thankless*`, `unfriendly*`, and `cruel*` are in scale 2.

```
> data(acl)
> Communality <- acl[, 1:10]
> scale <- aisp(Communality, verbose = FALSE)
> scale
```

	Scale
<code>reliable</code>	1
<code>honest</code>	1
<code>unscrupulous*</code>	0
<code>deceitful*</code>	1
<code>unintelligent*</code>	0
<code>obnoxious*</code>	2
<code>thankless*</code>	2
<code>unfriendly*</code>	2
<code>dependable</code>	1
<code>cruel*</code>	2

Variations of `aisp` (output not shown) are the following

- Use a genetic algorithm (Straat, van der Ark, & Sijtsma, 2010) rather than Mokken's algorithm.

```
> scale <- aisp(Communality, search = "ga")
```

- Use different values for the lower bound (default `lowerbound = .3`) and or the nominal type I error rate (default `alpha = .05`)

```
> scale <- aisp(Communality, lowerbound = 0.2, alpha = 0.1)
```

- Output on the screen during the item selection (default `verbose = TRUE`)

```
> aisp(Communality, verbose = TRUE)
```

- For more information type

```
> help(aisp)
```

Note that `search = "extend"` has not yet been implemented.

### 2.1.2 coefH

Computes scalability coefficients  $H_{ij}$ ,  $H_i$ , and  $H$  for a set of items. In the example, the scores on the first ten items from ACL are used; these are the items of the scale Communality. First, scalability coefficients  $H_{ij}$ ,  $H_i$ , and  $H$  are computed, plus their standard errors. The argument `se = FALSE` drops the standard errors. Second, only the item scalability coefficients are extracted. Third, the item scalability coefficients are extracted but rounded to two integers. Fourth,  $H$  is extracted. Fifth, the respondents are divided into two subgroups, those having a score less than 2 on item 11, and those having a score greater than or equal to 2 on item 11. Sixth, the scalability coefficients are computed for the entire sample, and for each of the two subsamples using the item-step ordering from the entire sample (this corresponds to the function `CHECK=GROUPS` in MSP). Seventh, the item-scalability coefficients are reported

```
> data(acl)
> Communality <- acl[, 1:10]
> coefficients <- coefH(Communality)
> coefficients$Hij
```

	reliable	se	honest	se
reliable			0.528	(0.052)
honest	0.528	(0.052)		



unscrupulous*	0.242	(0.068)	0.250	(0.071)
deceitful*	0.331	(0.055)	0.282	(0.057)
unintelligent*	0.132	(0.057)	0.139	(0.055)
obnoxious*	0.273	(0.059)	0.162	(0.057)
thankless*	0.183	(0.056)	0.160	(0.058)
unfriendly*	0.247	(0.061)	0.192	(0.074)
dependable	0.717	(0.046)	0.552	(0.054)
cruel*	0.116	(0.069)	0.141	(0.080)
	unscrupulous* se		deceitful* se	
reliable	0.242	(0.068)	0.331	(0.055)
honest	0.250	(0.071)	0.282	(0.057)
unscrupulous*			0.343	(0.059)
deceitful*	0.343	(0.059)		
unintelligent*	0.145	(0.060)	0.102	(0.053)
obnoxious*	0.239	(0.053)	0.321	(0.050)
thankless*	0.121	(0.051)	0.336	(0.054)
unfriendly*	0.241	(0.063)	0.399	(0.052)
dependable	0.229	(0.066)	0.317	(0.053)
cruel*	0.291	(0.061)	0.423	(0.055)
	unintelligent* se		obnoxious* se	
reliable	0.132	(0.057)	0.273	(0.059)
honest	0.139	(0.055)	0.162	(0.057)
unscrupulous*	0.145	(0.060)	0.239	(0.053)
deceitful*	0.102	(0.053)	0.321	(0.050)
unintelligent*			0.201	(0.052)
obnoxious*	0.201	(0.052)		
thankless*	0.133	(0.051)	0.339	(0.052)
unfriendly*	0.112	(0.056)	0.407	(0.052)
dependable	0.077	(0.059)	0.253	(0.058)
cruel*	0.005	(0.055)	0.372	(0.061)
	thankless* se		unfriendly* se	
reliable	0.183	(0.056)	0.247	(0.061)
honest	0.160	(0.058)	0.192	(0.074)
unscrupulous*	0.121	(0.051)	0.241	(0.063)
deceitful*	0.336	(0.054)	0.399	(0.052)
unintelligent*	0.133	(0.051)	0.112	(0.056)
obnoxious*	0.339	(0.052)	0.407	(0.052)
thankless*			0.430	(0.055)
unfriendly*	0.430	(0.055)		
dependable	0.227	(0.060)	0.254	(0.053)
cruel*	0.267	(0.066)	0.430	(0.065)
	dependable se		cruel* se	
reliable	0.717	(0.046)	0.116	(0.069)
honest	0.552	(0.054)	0.141	(0.080)

unscrupulous*	0.229	(0.066)	0.291	(0.061)
deceitful*	0.317	(0.053)	0.423	(0.055)
unintelligent*	0.077	(0.059)	0.005	(0.055)
obnoxious*	0.253	(0.058)	0.372	(0.061)
thankless*	0.227	(0.060)	0.267	(0.066)
unfriendly*	0.254	(0.053)	0.430	(0.065)
dependable			0.118	(0.077)
cruel*	0.118	(0.077)		

```
> coefficients$Hi
```

	Item H	se
reliable	0.304	(0.029)
honest	0.265	(0.034)
unscrupulous*	0.236	(0.034)
deceitful*	0.319	(0.028)
unintelligent*	0.116	(0.033)
obnoxious*	0.288	(0.028)
thankless*	0.245	(0.030)
unfriendly*	0.309	(0.032)
dependable	0.299	(0.032)
cruel*	0.252	(0.036)

```
> coefficients$H
```

Scale H	se
0.264	(0.020)

```
> subgroup <- ifelse(acl[, 11] < 2, 1, 2)
> coefficients <- coefH(Communality, group.var = subgroup)
> coefficients$Groups[[1]]$Hi
```

	Item H	se
reliable	0.269	(0.068)
honest	0.107	(0.075)
unscrupulous*	0.117	(0.070)
deceitful*	0.229	(0.072)
unintelligent*	0.130	(0.064)
obnoxious*	0.240	(0.056)
thankless*	0.170	(0.061)
unfriendly*	0.217	(0.064)
dependable	0.149	(0.078)
cruel*	0.208	(0.077)

```
> coefficients$Groups[[2]]$Hi
```

	Item H	se
reliable	0.313	(0.032)
honest	0.300	(0.037)
unscrupulous*	0.258	(0.038)
deceitful*	0.334	(0.030)
unintelligent*	0.108	(0.036)
obnoxious*	0.291	(0.032)
thankless*	0.255	(0.034)
unfriendly*	0.323	(0.036)
dependable	0.338	(0.032)
cruel*	0.258	(0.040)

### 2.1.3 check.iio

Investigates invariant item ordering (IIO) using method *Manifest IIO* (MIIO; Ligtvoet, Van der Ark, Te Marvelde, & Sijtsma, 2010) and methods *Manifest Scale - Cumulative Probability Model* (MS-CPM) and *Increasingness in Transposition* (IT) (Ligtvoet, Van der Ark, Bergsma, & Sijtsma, 2010). Method Manifest IIO is the default. First, all result with respect to IIO are saved in `iio.results`. In the example, the scores on the first ten items from ACL are used; these are the items of the scale Communalilty. Simply typing `iio.results` produces a list with lots of output for each item. `summary()` reduces this output by giving a summary of the results. The output shows the method used (i.e., Manifest IIO), the violations of manifest IIO, the items selected using the backward selection algorithm, and scalability coefficient  $H^T$  for the final scale (items `unfriendly*` and `deceitful*` excluded).

```
> data(acl)
> Communalilty <- acl[, 1:10]
> iio.results <- check.iio(Communalilty)
> summary(iio.results)

$method
[1] "MIIO"

$item.summary
      mean #ac #vi #vi/#ac maxvi  sum sum/#ac tmax
cruel*    3.48  27   0   0.00  0.00  0.00   0.00 0.00
unintelligent* 3.32  26   2   0.08  0.15 0.29   0.01 2.17
unscrupulous* 3.32  26   1   0.04  0.14 0.14   0.01 1.21
unfriendly*  3.30  27   1   0.04  0.15 0.15   0.01 2.17
thankless*   3.26  27   1   0.04  0.12 0.12   0.00 1.50
dependable   3.25  27   0   0.00  0.00 0.00   0.00 0.00
obnoxious*   3.25  27   1   0.04  0.12 0.12   0.00 1.50
```

reliable	3.09	27	0	0.00	0.00	0.00	0.00	0.00
honest	3.02	27	2	0.07	0.18	0.31	0.01	2.08
deceitful*	2.94	25	2	0.08	0.18	0.31	0.01	2.08

#tsig

cruel*	0
unintelligent*	1
unscrupulous*	0
unfriendly*	1
thankless*	0
dependable	0
obnoxious*	0
reliable	0
honest	1
deceitful*	1

\$backward.selection

	step 1	step 2	step 3
cruel*	0	0	0
unintelligent*	1	1	0
unscrupulous*	0	0	0
unfriendly*	1	1	NA
thankless*	0	0	0
dependable	0	0	0
obnoxious*	0	0	0
reliable	0	0	0
honest	1	0	0
deceitful*	1	NA	NA

\$HT

[1] 0.05807174

Variations of `check.iio` (output not shown) are the following.

- Other values for `minvi` and `minsize` (Molenaar & Sijtsma, 2000, pp. 45-46) .

```
> check.iio(Communality, minvi = 0, minsize = 50)
```

- Using methods MS-CPM and IT (Ligtvoet, Van der Ark, Bergsma, & Sijtsma, 2010)

```
> summary(check.iio(Communality, method = "MS-CPM"))
> summary(check.iio(Communality, method = "IT"))
```

- Different nominal Type I error rate for t-test (method MHIO), z-test (Method MS-CPM), and McNemar test (method IT).

```
> summary(check.iio(Communality, alpha = 0.01))
```

- Without backward selection algorithm, and with information screen

```
> summary(check.iio(Communality, item.selection = FALSE))
> summary(check.iio(Communality, verbose = TRUE))
```

- For more information type

```
> help(check.iio)
```

#### 2.1.4 check.monotonicity (a.k.a. check.single)

Investigates the monotonicity assumption using the observable property manifest monotonicity (Molenaar & Sijtsma, 2000, pp. 70-77). In the example, the scores on the first ten items from ACL are used; these are the items of the scale Communality. First, all result with respect to manifest monotonicity are saved in `monotonicity.results`. Simply typing `monotonicity.results` produces a list with lots of output for each item. `summary()` and `plot()` reduce this output by giving a summary of the results and graphically displaying the estimated item (step) response functions, respectively. For interpretation of the output see Molenaar and Sijtsma (2000, chap. 6, chap. 7). Without further specifications `plot()` displays 10 graphs (1 for each item) in a separate R Window, and requires a hard return to go to the next graph. Figure 2.1.4 (p. 22) shows the 10 graphs.

```
> data(acl)
> Communality <- acl[, 1:10]
> monotonicity.results <- check.monotonicity(Communality)
> summary(monotonicity.results)
> plot(monotonicity.results, items = c(1, 2))
```

	ItemH	#ac	#vi	#vi/#ac	maxvi	sum	sum/#ac
reliable	0.30	15	0	0.00	0.00	0.00	0
honest	0.27	16	0	0.00	0.00	0.00	0
unscrupulous*	0.24	20	0	0.00	0.00	0.00	0
deceitful*	0.32	22	0	0.00	0.00	0.00	0
unintelligent*	0.12	20	1	0.05	0.07	0.07	0
obnoxious*	0.29	19	0	0.00	0.00	0.00	0
thankless*	0.25	17	0	0.00	0.00	0.00	0
unfriendly*	0.31	19	0	0.00	0.00	0.00	0
dependable	0.30	17	0	0.00	0.00	0.00	0
cruel*	0.25	19	0	0.00	0.00	0.00	0
	zmax	#zsig					
reliable	0.00	0					

honest	0.00	0
unscrupulous*	0.00	0
deceitful*	0.00	0
unintelligent*	0.85	0
obnoxious*	0.00	0
thankless*	0.00	0
unfriendly*	0.00	0
dependable	0.00	0
cruel*	0.00	0

Variations of `check.monotonicity` (output not shown) are the following.

- Other values for `minvi` and `minsize` (Molenaar & Sijtsma, 2000, pp. 45-46) .

```
> check.monotonicity(Communality, minvi = 0, minsize = 50)
```

- Plot the results for items 1 and 2 only

```
> plot(check.monotonicity(Communality), item = c(1,
+      2))
```

- Save graphs in a pdf file. `ask=FALSE` assures that no hard return is required between subsequent graphs. The functions `pdf()` and `dev.off()` are not part of `mokken`.

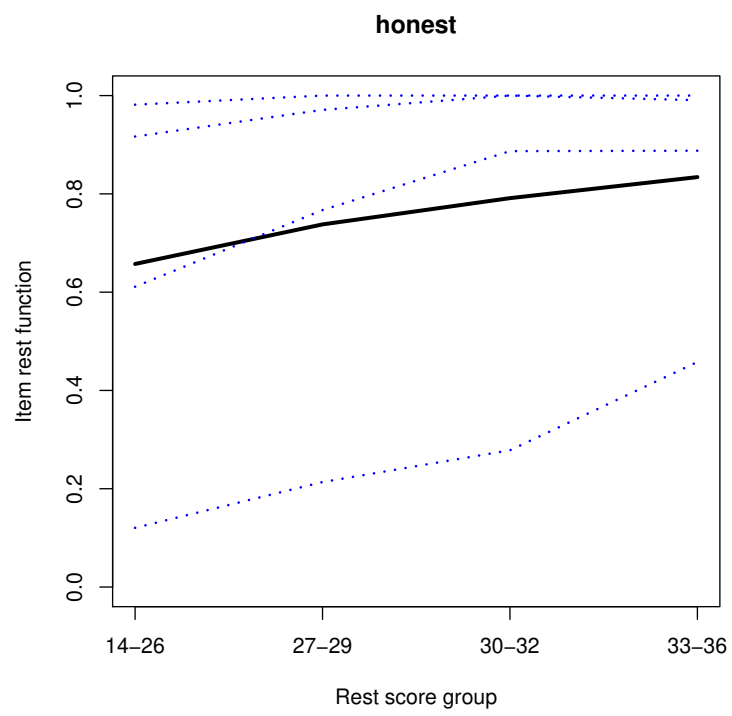
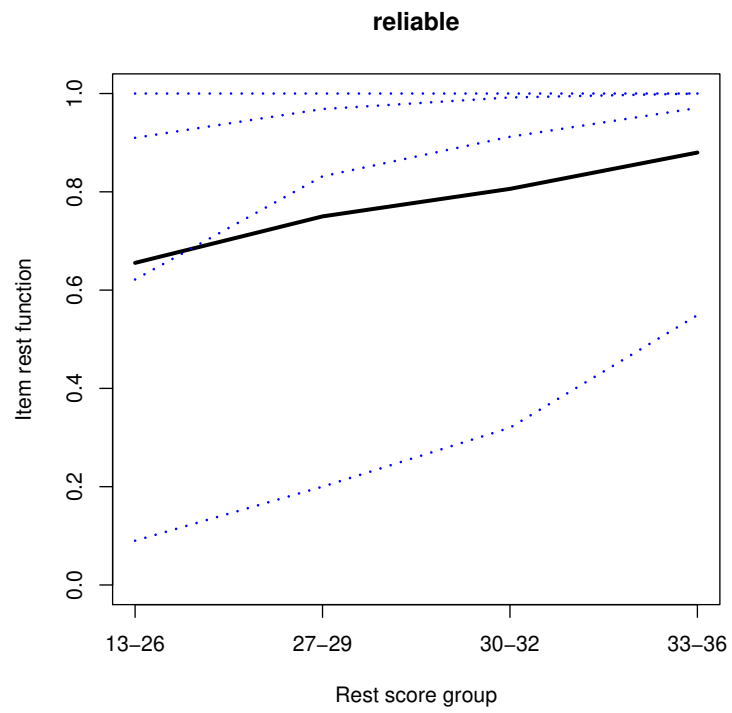
```
> pdf("monotonicity.pdf")
> plot(monotonicity.results, ask = FALSE)
> dev.off()
```

- For more information type

```
> help(check.monotonicity)
```

### 2.1.5 check.pmatrix

Investigates the assumption of nonintersecting item step response functions using the P++ and P-- matrix (Molenaar & Sijtsma, 2000, pp. 80-85). In the example, the scores on the first ten items from ACL are used; these are the items of the scale `Communality`. First, all result with respect to the P++ (indicted by `ppp`) and P-- (indicated by `pmm`) matrix are saved in `pmatrix.results`. Simply typing `pmatrix.results` produces a list with lots of output for each item. `summary()` and `plot()` reduce this output by giving a summary of the results and graphically displaying the estimated item (step) response functions, respectively. For interpretation of the output



see Molenaar and Sijtsma (2000, pp. 80-85). Without further specifications `plot()` (no output shown) displays 20 graphs (2 for each item) in a separate R Window, and requires a hard return to go to the next graph.

```
> data(acl)
> Communality <- acl[, 1:10]
> pmatrix.results <- check.pmatrix(Communality)
> summary(pmatrix.results)
> plot(pmatrix.results)

> pmatrix.results <- check.pmatrix(Communality)
> summary(pmatrix.results)
```

	Hi	#ac	#vi	#vi/#ac	maxvi	sum	sum/#ac
reliable	0.30	4608	22	0.00	0.05	0.85	2e-04
honest	0.27	4608	3	0.00	0.04	0.10	0e+00
unscrupulous*	0.24	4608	6	0.00	0.05	0.23	1e-04
deceitful*	0.32	4608	8	0.00	0.07	0.33	1e-04
unintelligent*	0.12	4608	12	0.00	0.08	0.50	1e-04
obnoxious*	0.29	4608	7	0.00	0.08	0.31	1e-04
thankless*	0.25	4608	7	0.00	0.08	0.33	1e-04
unfriendly*	0.31	4608	9	0.00	0.08	0.39	1e-04
dependable	0.30	4608	24	0.01	0.08	1.14	2e-04
cruel*	0.25	4608	10	0.00	0.06	0.37	1e-04

	zmax	#zsig	crit
reliable	4.27	19	84
honest	2.75	3	41
unscrupulous*	4.06	6	60
deceitful*	3.24	7	57
unintelligent*	4.55	10	82
obnoxious*	5.12	7	69
thankless*	4.75	6	67
unfriendly*	4.45	8	67
dependable	5.12	23	98
cruel*	4.40	9	69

Variations of `check.pmatrix` are the following.

- Other values for `minvi` (Molenaar & Sijtsma, 2000, pp. 45-46) .

```
> check.pmatrix(Communality, minvi = 0)
```

- Plot the results for P++, for items 1 and 2 only, and plot the results for P-- for item 5.



```
> plot(check.pmatrix(Communality), pmatrix = "ppp",
+      items = c(1, 2))
> plot(check.pmatrix(Communality), pmatrix = "pmm",
+      items = 5)
```

- Save graphs in a pdf file. `ask=FALSE` assures that no hard return is required between subsequent graphs. The functions `pdf()` and `dev.off()` are not part of `mokken`.

```
> pdf("pmatrix.pdf")
> plot(pmatrix.results, ask = FALSE)
> dev.off()
```

- For more information type

```
> help(check.pmatrix)
```

### 2.1.6 check.reliability

Computes reliability coefficients  $\rho$  (a.k.a., the MS statistic; Molenaar & Sijtsma, 1984, 1988; Sijtsma & Molenaar, 1987; Van der Ark, 2010), Cronbach's (1951) alpha, lambda-2 (Guttman, 1945), and on request (not default) the latent class reliability coefficient (LCRC, Van der Ark, Van der Palm, & Sijtsma, 2011). In the example, the scores on the first ten items from ACL are used; these are the items of the scale `Communality`.

```
> data(acl)
> Communality <- acl[, 1:10]
> check.reliability(Communality)
```

```
$MS
[1] 0.75766
```

```
$alpha
[1] 0.7465871
```

```
$lambda.2
[1] 0.7568063
```

### 2.1.7 check.restscore

Investigates the assumption of nonintersecting item step response functions using method `restscore` (Molenaar & Sijtsma, 2000, pp. 77-80). In the example, the scores on the first ten items from ACL are used; these are the items of the scale `Communality`. First, all result with respect to method `restscore`

are saved in `restscore.results`. Simply typing `restscore.results` produces a list with lots of output for each item pair. `summary()` and `plot()` reduce this output by giving a summary of the results and plotting the estimated item (step) response functions, respectively. For interpretation of the output see Molenaar and Sijtsma (2000, pp. 77-80). Without further specifications `plot()` displays  $\frac{1}{2} \times 10 \times 9 = 45$  graphs (1 for each item pair) in a separate R Window, and requires a hard return to go to the next graph. Figure 2.1.7 (p. 26) shows the rest score plots for the first two item pairs.

```
> data(ac1)
> Communality <- ac1[, 1:10]
> restscore.results <- check.restscore(Communality)
> summary(restscore.results)
> plot(restscore.results, item.pairs = c(1, 2))
```

	ItemH	#ac	#vi	#vi/#ac	maxvi	sum	sum/#ac
reliable	0.30	432	7	0.02	0.09	0.31	0
honest	0.27	432	5	0.01	0.07	0.25	0
unscrupulous*	0.24	416	6	0.01	0.11	0.42	0
deceitful*	0.32	400	8	0.02	0.09	0.40	0
unintelligent*	0.12	416	14	0.03	0.11	0.86	0
obnoxious*	0.29	432	7	0.02	0.11	0.52	0
thankless*	0.25	432	7	0.02	0.08	0.38	0
unfriendly*	0.31	432	8	0.02	0.11	0.48	0
dependable	0.30	432	9	0.02	0.09	0.48	0
cruel*	0.25	432	3	0.01	0.04	0.12	0

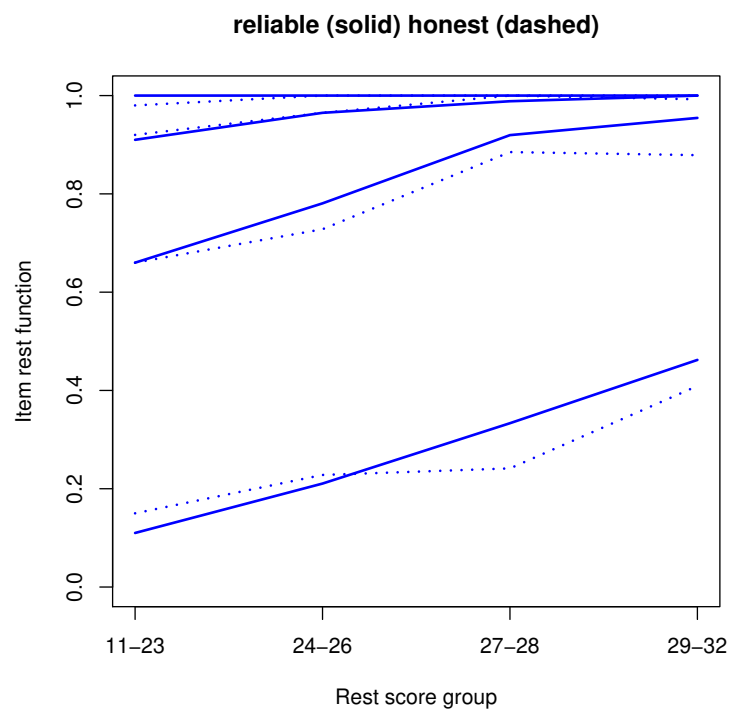
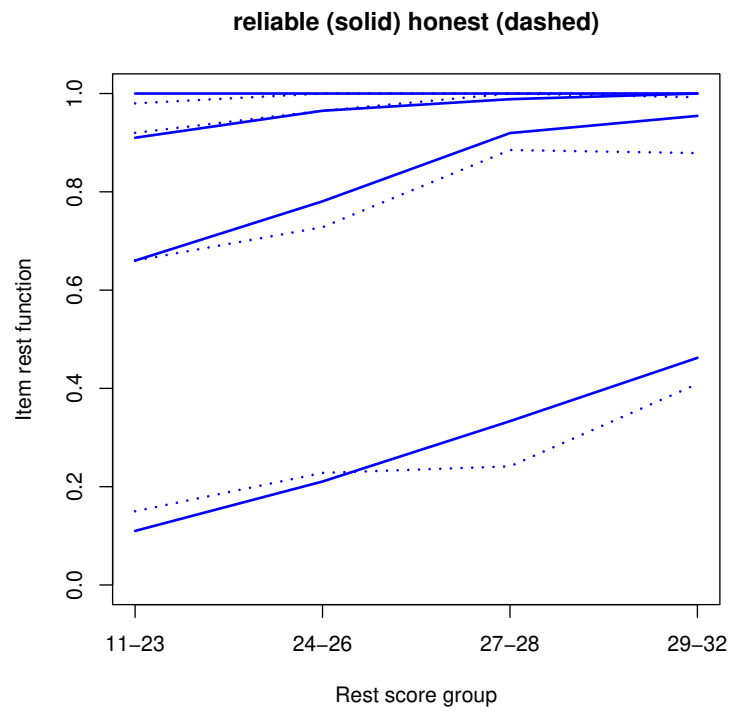
  

	zmax	#zsig	crit
reliable	1.43	0	26
honest	1.25	0	23
unscrupulous*	1.46	0	33
deceitful*	1.22	0	26
unintelligent*	1.99	2	62
obnoxious*	1.83	1	44
thankless*	1.14	0	27
unfriendly*	1.99	1	43
dependable	1.22	0	28
cruel*	0.67	0	16

Variations of `check.restscore` are the following.

- Other values for `minvi` and `minsize` (Molenaar & Sijtsma, 2000, pp. 45-46) .

```
> check.restscore(Communality, minvi = 0, minsize = 50)
```



- Plot the results for all item pairs.

```
> plot(check.restscore(Communality))
```

- Save graphs in a pdf file. `ask=FALSE` assures that no hard return is required between subsequent graphs. The functions `pdf()` and `dev.off()` are not part of `mokken`.

```
> pdf("restscore.pdf")
> plot(restscore.results, ask = FALSE)
> dev.off()
```

- For more information type

```
> help(check.restscore)
```

### 2.1.8 check.groups

The package `mokken` does not have a function `check.groups`, which—in analogy to the function `CHECK=GROUPS` in the software program MSP (Moleenaar & Sijtsma, 2000, pp. 85-88)—may have been expected.

Some analyses can be conducted using the command `coefH`.

```
> data(acl)
> Communality <- acl[, 1:10]
> subgroup <- ifelse(acl[, 11] < 2, 1, 2)
> coefH(Communality, group.var = subgroup)
```

\$Hij

	reliable	se	honest	se
reliable			0.528	(0.052)
honest	0.528	(0.052)		
unscrupulous*	0.242	(0.068)	0.250	(0.071)
deceitful*	0.331	(0.055)	0.282	(0.057)
unintelligent*	0.132	(0.057)	0.139	(0.055)
obnoxious*	0.273	(0.059)	0.162	(0.057)
thankless*	0.183	(0.056)	0.160	(0.058)
unfriendly*	0.247	(0.061)	0.192	(0.074)
dependable	0.717	(0.046)	0.552	(0.054)
cruel*	0.116	(0.069)	0.141	(0.080)
	unscrupulous*	se	deceitful*	se
reliable	0.242	(0.068)	0.331	(0.055)
honest	0.250	(0.071)	0.282	(0.057)
unscrupulous*			0.343	(0.059)
deceitful*	0.343	(0.059)		

unintelligent*	0.145	(0.060)	0.102	(0.053)
obnoxious*	0.239	(0.053)	0.321	(0.050)
thankless*	0.121	(0.051)	0.336	(0.054)
unfriendly*	0.241	(0.063)	0.399	(0.053)
dependable	0.229	(0.066)	0.317	(0.053)
cruel*	0.291	(0.061)	0.423	(0.055)
	unintelligent* se		obnoxious* se	
reliable	0.132	(0.057)	0.273	(0.059)
honest	0.139	(0.055)	0.162	(0.057)
unscrupulous*	0.145	(0.060)	0.239	(0.053)
deceitful*	0.102	(0.053)	0.321	(0.050)
unintelligent*			0.201	(0.052)
obnoxious*	0.201	(0.052)		
thankless*	0.133	(0.051)	0.339	(0.052)
unfriendly*	0.112	(0.056)	0.407	(0.052)
dependable	0.077	(0.059)	0.253	(0.058)
cruel*	0.005	(0.055)	0.372	(0.061)
	thankless* se		unfriendly* se	
reliable	0.183	(0.056)	0.247	(0.061)
honest	0.160	(0.058)	0.192	(0.074)
unscrupulous*	0.121	(0.051)	0.241	(0.063)
deceitful*	0.336	(0.054)	0.399	(0.053)
unintelligent*	0.133	(0.051)	0.112	(0.056)
obnoxious*	0.339	(0.052)	0.407	(0.052)
thankless*			0.430	(0.055)
unfriendly*	0.430	(0.055)		
dependable	0.227	(0.060)	0.254	(0.053)
cruel*	0.267	(0.066)	0.430	(0.065)
	dependable se		cruel* se	
reliable	0.717	(0.046)	0.116	(0.069)
honest	0.552	(0.054)	0.141	(0.080)
unscrupulous*	0.229	(0.066)	0.291	(0.061)
deceitful*	0.317	(0.053)	0.423	(0.055)
unintelligent*	0.077	(0.059)	0.005	(0.055)
obnoxious*	0.253	(0.058)	0.372	(0.061)
thankless*	0.227	(0.060)	0.267	(0.066)
unfriendly*	0.254	(0.053)	0.430	(0.065)
dependable			0.118	(0.077)
cruel*	0.118	(0.077)		

\$Hi

	Item H	se
reliable	0.304	(0.029)
honest	0.265	(0.034)

unscrupulous*	0.236 (0.034)
deceitful*	0.319 (0.028)
unintelligent*	0.116 (0.033)
obnoxious*	0.288 (0.028)
thankless*	0.245 (0.030)
unfriendly*	0.309 (0.032)
dependable	0.299 (0.032)
cruel*	0.252 (0.036)

\$H

Scale H se

0.264 (0.020)

\$Groups

\$Groups\$`1`

\$Groups\$`1`\$Hij

	reliable se	honest se		
reliable		0.510 (0.121)		
honest	0.510 (0.121)			
unscrupulous*	0.367 (0.153)	0.220 (0.130)		
deceitful*	0.275 (0.139)	0.234 (0.151)		
unintelligent*	0.079 (0.129)	-0.069 (0.125)		
obnoxious*	0.335 (0.121)	-0.048 (0.101)		
thankless*	0.099 (0.133)	-0.054 (0.115)		
unfriendly*	0.191 (0.123)	-0.095 (0.133)		
dependable	0.583 (0.147)	0.381 (0.134)		
cruel*	0.079 (0.186)	-0.039 (0.175)		
	unscrupulous* se	deceitful* se		
reliable	0.367 (0.153)	0.275 (0.139)		
honest	0.220 (0.130)	0.234 (0.151)		
unscrupulous*		0.024 (0.133)		
deceitful*	0.024 (0.133)			
unintelligent*	-0.130 (0.156)	0.192 (0.121)		
obnoxious*	0.131 (0.153)	0.233 (0.127)		
thankless*	0.104 (0.098)	0.169 (0.136)		
unfriendly*	0.079 (0.116)	0.214 (0.111)		
dependable	0.194 (0.156)	0.282 (0.158)		
cruel*	0.087 (0.122)	0.422 (0.126)		
	unintelligent* se	obnoxious* se		
reliable	0.079 (0.129)	0.335 (0.121)		
honest	-0.069 (0.125)	-0.048 (0.101)		
unscrupulous*	-0.130 (0.156)	0.131 (0.153)		
deceitful*	0.192 (0.121)	0.233 (0.127)		
unintelligent*		0.254 (0.127)		

obnoxious*	0.254	(0.127)		
thankless*	0.205	(0.137)	0.278	(0.142)
unfriendly*	0.143	(0.141)	0.475	(0.115)
dependable	0.100	(0.110)	0.008	(0.124)
cruel*	0.323	(0.118)	0.492	(0.124)
	thankless* se		unfriendly* se	
reliable	0.099	(0.133)	0.191	(0.123)
honest	-0.054	(0.115)	-0.095	(0.133)
unscrupulous*	0.104	(0.098)	0.079	(0.116)
deceitful*	0.169	(0.136)	0.214	(0.111)
unintelligent*	0.205	(0.137)	0.143	(0.141)
obnoxious*	0.278	(0.142)	0.475	(0.115)
thankless*			0.491	(0.123)
unfriendly*	0.491	(0.123)		
dependable	0.057	(0.132)	0.051	(0.099)
cruel*	0.224	(0.171)	0.366	(0.150)
	dependable se		cruel* se	
reliable	0.583	(0.147)	0.079	(0.186)
honest	0.381	(0.134)	-0.039	(0.175)
unscrupulous*	0.194	(0.156)	0.087	(0.122)
deceitful*	0.282	(0.158)	0.422	(0.126)
unintelligent*	0.100	(0.110)	0.323	(0.118)
obnoxious*	0.008	(0.124)	0.492	(0.124)
thankless*	0.057	(0.132)	0.224	(0.171)
unfriendly*	0.051	(0.099)	0.366	(0.150)
dependable			-0.192	(0.144)
cruel*	-0.192	(0.144)		

\$Groups\$`1`\$Hi

	Item H	se
reliable	0.269	(0.068)
honest	0.107	(0.075)
unscrupulous*	0.117	(0.070)
deceitful*	0.229	(0.072)
unintelligent*	0.129	(0.064)
obnoxious*	0.239	(0.056)
thankless*	0.170	(0.061)
unfriendly*	0.215	(0.063)
dependable	0.149	(0.078)
cruel*	0.208	(0.077)

\$Groups\$`1`\$H

Scale H	se
0.185	(0.044)

\$Groups\$`2`  
\$Groups\$`2`\$Hij

	reliable	se	honest	se
reliable			0.532	(0.057)
honest	0.532	(0.057)		
unscrupulous*	0.216	(0.075)	0.255	(0.080)
deceitful*	0.345	(0.059)	0.293	(0.061)
unintelligent*	0.144	(0.062)	0.179	(0.060)
obnoxious*	0.259	(0.065)	0.211	(0.065)
thankless*	0.208	(0.061)	0.221	(0.065)
unfriendly*	0.266	(0.067)	0.270	(0.078)
dependable	0.744	(0.043)	0.586	(0.057)
cruel*	0.126	(0.072)	0.182	(0.088)
	unscrupulous*	se	deceitful*	se
reliable	0.216	(0.075)	0.345	(0.059)
honest	0.255	(0.080)	0.293	(0.061)
unscrupulous*			0.404	(0.064)
deceitful*	0.404	(0.064)		
unintelligent*	0.182	(0.065)	0.077	(0.058)
obnoxious*	0.256	(0.057)	0.330	(0.055)
thankless*	0.122	(0.059)	0.366	(0.059)
unfriendly*	0.272	(0.072)	0.434	(0.061)
dependable	0.238	(0.073)	0.328	(0.055)
cruel*	0.331	(0.067)	0.419	(0.061)
	unintelligent*	se	obnoxious*	se
reliable	0.144	(0.062)	0.259	(0.065)
honest	0.179	(0.060)	0.211	(0.065)
unscrupulous*	0.182	(0.065)	0.256	(0.057)
deceitful*	0.077	(0.058)	0.330	(0.055)
unintelligent*			0.182	(0.056)
obnoxious*	0.182	(0.056)		
thankless*	0.105	(0.053)	0.332	(0.057)
unfriendly*	0.091	(0.060)	0.378	(0.059)
dependable	0.077	(0.067)	0.319	(0.062)
cruel*	-0.061	(0.057)	0.340	(0.067)
	thankless*	se	unfriendly*	se
reliable	0.208	(0.061)	0.266	(0.067)
honest	0.221	(0.065)	0.270	(0.078)
unscrupulous*	0.122	(0.059)	0.272	(0.072)
deceitful*	0.366	(0.059)	0.434	(0.061)
unintelligent*	0.105	(0.053)	0.091	(0.060)
obnoxious*	0.332	(0.057)	0.378	(0.059)



thankless*			0.397	(0.063)
unfriendly*	0.397	(0.063)		
dependable	0.282	(0.065)	0.317	(0.058)
cruel*	0.267	(0.070)	0.439	(0.071)
	dependable	se	cruel*	se
reliable	0.744	(0.043)	0.126	(0.072)
honest	0.586	(0.057)	0.182	(0.088)
unscrupulous*	0.238	(0.073)	0.331	(0.067)
deceitful*	0.328	(0.055)	0.419	(0.061)
unintelligent*	0.077	(0.067)	-0.061	(0.057)
obnoxious*	0.319	(0.062)	0.340	(0.067)
thankless*	0.282	(0.065)	0.267	(0.070)
unfriendly*	0.317	(0.058)	0.439	(0.071)
dependable			0.196	(0.081)
cruel*	0.196	(0.081)		

\$Groups\$`2`\$Hi

	Item	H	se
reliable	0.313	(0.032)	
honest	0.300	(0.037)	
unscrupulous*	0.258	(0.038)	
deceitful*	0.334	(0.030)	
unintelligent*	0.108	(0.037)	
obnoxious*	0.291	(0.032)	
thankless*	0.255	(0.034)	
unfriendly*	0.323	(0.036)	
dependable	0.338	(0.032)	
cruel*	0.258	(0.040)	

\$Groups\$`2`\$H

Scale	H	se
	0.277	(0.022)

## Chapter 3

# Examples of Mokken scale analysis in R

This chapter shows the code for producing the tables in Sijtsma and Molenaar (2003).

### 3.1 Table 3.1

Get the transitive reasoning data, and split them into the grades (first column of the data matrix), and the items scores (the remaining columns in the data matrix).

```
> library(mokken)
> data(transreas)
> grades <- transreas[, 1]
> item.scores <- transreas[, -1]
```

Obtaining the overall mean scores, and the mean scores per grade

```
> apply(item.scores, 2, mean)
```

	T09L	T12P	T10W	T11P	T04W	T05W
	0.3011765	0.4752941	0.5200000	0.6423529	0.7835294	0.8023529
	T02L	T07L	T03W	T01L	T08W	T06A
	0.8094118	0.8447059	0.8847059	0.9411765	0.9670588	0.9741176

```
> apply(item.scores[grades == 2, ], 2, mean)
```

	T09L	T12P	T10W	T11P	T04W	T05W
	0.2093023	0.6627907	0.4069767	0.4883721	0.8372093	0.7093023
	T02L	T07L	T03W	T01L	T08W	T06A
	0.8139535	0.7209302	0.8023256	0.8488372	0.9302326	0.9534884

```
> apply(item.scores[grades == 3, ], 2, mean)
```

T09L	T12P	T10W	T11P	T04W	T05W
0.3294118	0.5058824	0.5764706	0.6000000	0.6470588	0.8000000
T02L	T07L	T03W	T01L	T08W	T06A
0.7294118	0.8352941	0.9294118	0.9882353	0.9647059	0.9529412

```
> apply(item.scores[grades == 4, ], 2, mean)
```

T09L	T12P	T10W	T11P	T04W	T05W
0.3048780	0.3902439	0.3902439	0.6707317	0.8170732	0.7804878
T02L	T07L	T03W	T01L	T08W	T06A
0.8902439	0.8658537	0.9146341	0.9756098	0.9756098	0.9878049

```
> apply(item.scores[grades == 5, ], 2, mean)
```

T09L	T12P	T10W	T11P	T04W	T05W
0.2588235	0.5176471	0.5529412	0.7058824	0.8235294	0.8235294
T02L	T07L	T03W	T01L	T08W	T06A
0.7882353	0.8705882	0.8470588	0.9294118	0.9764706	0.9764706

```
> apply(item.scores[grades == 6, ], 2, mean)
```

T09L	T12P	T10W	T11P	T04W	T05W
0.4022989	0.2988506	0.6666667	0.7471264	0.7931034	0.8965517
T02L	T07L	T03W	T01L	T08W	T06A
0.8275862	0.9310345	0.9310345	0.9655172	0.9885057	1.0000000

Construction of Table 3.1 (advanced R code).

```
> Total.group <- round(apply(item.scores, 2, mean),
+ 2)
> for (i in 2:6) assign(paste("Grade.", i, sep = ""),
+ round(apply(item.scores[grades == i, ], 2,
+ mean), 2))
> Task <- c(9, 12, 10, 11, 4, 5, 2, 7, 3, 1, 8,
+ 6)
> Property <- attributes(transreas)$property
> Format <- attributes(transreas)$format
> Objects <- attributes(transreas)$objects
> Measures <- attributes(transreas)$measures
> Table.3.1 <- data.frame(Task, Property, Format,
+ Objects, Measures, Total.group, Grade.2, Grade.3,
+ Grade.4, Grade.5, Grade.6)
> Table.3.1
```

	Task	Property	Format	Objects		
T09L	9	length	YA = YB < YC = YD	sticks		
T12P	12	pseudo				
T10W	10	weight	YA = YB < YC = YD	balls		
T11P	11	pseudo				
T04W	4	weight	YA = YB = YC = YD	cubes		
T05W	5	weight	YA < YB < YC	balls		
T02L	2	length	YA = YB = YC = YD	tubes		
T07L	7	length	YA > YB = YC	sticks		
T03W	3	weight	YA > YB > YC	tubes		
T01L	1	length	YA > YB > YC	sticks		
T08W	8	weight	YA > YB = YC	balls		
T06A	6	area	YA > YB > YC	discs		
			Measures	Total.group	Grade.2	Grade.3
T09L		12.5, 12.5, 13, 13 (cm)		0.30	0.21	0.33
T12P				0.48	0.66	0.51
T10W		60, 60, 100, 100 (g)		0.52	0.41	0.58
T11P				0.64	0.49	0.60
T04W		65 (g)		0.78	0.84	0.65
T05W		40, 50, 70 (cm)		0.80	0.71	0.80
T02L		12 (cm)		0.81	0.81	0.73
T07L		28.5, 27.5, 27.5 (cm)		0.84	0.72	0.84
T03W		45, 25, 18 (g)		0.88	0.80	0.93
T01L		12, 11.5, 11 (cm)		0.94	0.85	0.99
T08W		65, 40, 40 (g)		0.97	0.93	0.96
T06A		7.5, 7, 6.5 (diameter; cm)		0.97	0.95	0.95
		Grade.4	Grade.5	Grade.6		
T09L		0.30	0.26	0.40		
T12P		0.39	0.52	0.30		
T10W		0.39	0.55	0.67		
T11P		0.67	0.71	0.75		
T04W		0.82	0.82	0.79		
T05W		0.78	0.82	0.90		
T02L		0.89	0.79	0.83		
T07L		0.87	0.87	0.93		
T03W		0.91	0.85	0.93		
T01L		0.98	0.93	0.97		
T08W		0.98	0.98	0.99		
T06A		0.99	0.98	1.00		

## 3.2 Table 3.2

To get the data, see Table 3.1.

Obtain scalability coefficients and  $Z$  coefficients for items and total scale.

```
> coefH(item.scores, FALSE)$Hi
```

	T09L	T12P	T10W	T11P	T04W
	0.17075112	-0.13516424	0.13762914	-0.02534203	0.04791651
	T05W	T02L	T07L	T03W	T01L
	0.09166970	0.08126425	0.17766194	0.18791305	0.20624790
	T08W	T06A			
	0.28245200	0.39559408			

```
> coefH(item.scores, FALSE)$H
```

```
[1] 0.090514
```

```
> coefZ(item.scores)$Zi
```

	T09L	T12P	T10W	T11P	T04W	T05W
	4.357153	-4.650737	4.949901	-0.967124	1.948629	3.746486
	T02L	T07L	T03W	T01L	T08W	T06A
	3.307606	6.868759	6.582933	5.684119	6.326821	8.019603

```
> coefZ(item.scores)$Z
```

```
[1] 7.395366
```

Obtain scalability coefficients and  $Z$  coefficients for items and total scale, when the pseudo items (2 and 4) are deleted

```
> coefH(item.scores[, -c(2, 4)], FALSE)$Hi
```

	T09L	T10W	T04W	T05W	T02L
	0.28926218	0.28083167	0.03625403	0.13900577	0.07160711
	T07L	T03W	T01L	T08W	T06A
	0.24977935	0.29285894	0.28657074	0.38746871	0.48272634

```
> coefH(item.scores[, -c(2, 4)], FALSE)$H
```

```
[1] 0.2048305
```

```
> coefZ(item.scores[, -c(2, 4)])$Zi
```

	T09L	T10W	T04W	T05W	T02L	T07L
	5.548097	7.506324	1.347390	5.285802	2.725229	9.192316
	T03W	T01L	T08W	T06A		
	9.922119	7.821882	8.732480	9.881951		

```
> coefZ(item.scores[, -c(2, 4)])$Z
```

```
[1] 13.54854
```

Construction of Table 3.2 (advanced R code).

```
> Task <- c("9", "12", "10", "11", "4", "5", "2",
+ "7", "3", "1", "8", "6", "Total item set")
> Property <- c(attributes(transreas)$property,
+ "")
> Format <- c(attributes(transreas)$format, "")
> Table.3.2 <- data.frame(Task, Property, Format,
+ matrix(NA, 13, 8))
> analysis <- list(c(1:12), c(1, 3, 5:12), c(1,
+ 3, 6, 8:12), c(1, 3, 8:12))
> k <- 3
> for (i in 1:4) for (j in 1:2) {
+   k <- k + 1
+   Table.3.2[c(analysis[[i]], 13), k] <- c(round(coefH(item.scores[,
+ analysis[[i]]], FALSE)$Hi, 2), round(coefH(item.scores[,
+ analysis[[i]]], FALSE)$H, 2))
+ }
> dimnames(Table.3.2)[[2]][4:11] <- paste(c("k=12",
+ "k=12", "k=10", "k=10", "k=8", "k=8", "k=7",
+ "k=7"), c("Hi", "Zi"))
> Table.3.2
```

	Task	Property	Format	k=12	Hi									
1	9	length	YA = YB < YC = YD		0.17									
2	12	pseudo			-0.14									
3	10	weight	YA = YB < YC = YD		0.14									
4	11	pseudo			-0.03									
5	4	weight	YA = YB = YC = YD		0.05									
6	5	weight	YA < YB < YC		0.09									
7	2	length	YA = YB = YC = YD		0.08									
8	7	length	YA > YB = YC		0.18									
9	3	weight	YA > YB > YC		0.19									
10	1	length	YA > YB > YC		0.21									
11	8	weight	YA > YB = YC		0.28									
12	6	area	YA > YB > YC		0.40									
13	Total item set					0.09								
	k=12	Zi	k=10	Hi	k=10	Zi	k=8	Hi	k=8	Zi	k=7	Hi	k=7	Zi
1	0.17		0.29		0.29		0.44		0.44		0.50		0.50	
2	-0.14		NA		NA		NA		NA		NA		NA	

3	0.14	0.28	0.28	0.46	0.46	0.52	0.52
4	-0.03	NA	NA	NA	NA	NA	NA
5	0.05	0.04	0.04	NA	NA	NA	NA
6	0.09	0.14	0.14	0.20	0.20	NA	NA
7	0.08	0.07	0.07	NA	NA	NA	NA
8	0.18	0.25	0.25	0.39	0.39	0.51	0.51
9	0.19	0.29	0.29	0.45	0.45	0.53	0.53
10	0.21	0.29	0.29	0.42	0.42	0.46	0.46
11	0.28	0.39	0.39	0.51	0.51	0.55	0.55
12	0.40	0.48	0.48	0.57	0.57	0.59	0.59
13	0.09	0.20	0.20	0.40	0.40	0.52	0.52

### 3.3 Table 5.1

To get the data, see Table 3.1.

Automated item selection algorithm

```
> options(width = 60)
> scale <- aisp(item.scores, verbose = FALSE)
```

Construction of Table 5.1 (advanced R code).

```
> options(width = 60)
> scale.1 <- c(12, 8, 1, 11, 9, 3, 10)
> scale.2 <- c(7, 5)
> Hi.top <- matrix(NA, 8, 6)
> for (i in 1:6) Hi.top[1:(i + 1), i] <- round(coefH(item.scores[,
+   scale.1[1:(i + 1)]], FALSE)$Hi, 2)
> for (i in 1:6) Hi.top[8, i] <- round(coefH(item.scores[,
+   scale.1[1:(i + 1)]], FALSE)$H, 2)
> dimnames(Hi.top)[[2]] <- paste("Step", 1:6)
> Table.5.1.top <- data.frame(Task = c(Task[scale.1],
+   "Total H"), Property = c(Property[scale.1],
+   ""), Format = c(Format[scale.1], ""), Pi = c(round(apply(item.scores[,
+   scale.1], 2, mean), 2), NA))
> Table.5.1.top <- cbind(Table.5.1.top, Hi.top)
> Table.5.1.top
```

	Task	Property	Format	Pi	Step 1	Step 2
T06A	6	area	YA > YB > YC	0.97	0.78	0.76
T07L	7	length	YA > YB = YC	0.84	0.78	0.59
T09L	9	length	YA = YB < YC = YD	0.30	NA	0.53
T08W	8	weight	YA > YB = YC	0.97	NA	NA

T03W	3	weight	YA > YB > YC	0.88	NA	NA
T10W	10	weight	YA = YB < YC = YD	0.52	NA	NA
T01L	1	length	YA > YB > YC	0.94	NA	NA
Total H				NA	0.78	0.60
Step 3 Step 4 Step 5 Step 6						
T06A	0.66	0.64	0.64	0.59		
T07L	0.56	0.50	0.51	0.51		
T09L	0.60	0.57	0.51	0.50		
T08W	0.59	0.62	0.61	0.55		
T03W	NA	0.51	0.53	0.53		
T10W	NA	NA	0.52	0.52		
T01L	NA	NA	NA	0.46		
	0.59	0.55	0.53	0.52		

### 3.4 Table 5.2

Get the data, and dichotomize the scores, compute the *P*-values

```
> data(cavalini)
> X <- cavalini
> X[cavalini < 2] <- 0
> X[cavalini > 1] <- 1
> apply(X, 2, mean)
```

Item1	Item2	Item3	Item4	Item5
0.60024155	0.43478261	0.59057971	0.42632850	0.20289855
Item6	Item7	Item8	Item9	Item10
0.08937198	0.04710145	0.16062802	0.06280193	0.51932367
Item11	Item12	Item13	Item14	Item15
0.21618357	0.28623188	0.13647343	0.22342995	0.16304348
Item16	Item17			
0.16666667	0.68599034			

Make the table (advanced R code)

```
> Table.5.2 <- data.frame(1:17, attributes(X)$labels,
+   round(apply(X, 2, mean), 2))
> dimnames(Table.5.2)[[2]] <- c("Item.number", "Item.text",
+   "Pi")
> rownames(Table.5.2) <- NULL
> Table.5.2
```

Item.number	Item.text	Pi
1	1	Keep windows closed 0.60



2	2	no laundry outside	0.43
3	3	search source of malodour	0.59
4	4	no blankets outside	0.43
5	5	try to find out solutions	0.20
6	6	go elsewhere to fresh air	0.09
7	7	call environment agency	0.05
8	8	think of something else	0.16
9	9	file complaint at producer	0.06
10	10	acquiesce in odour annoyance	0.52
11	11	do something to get rid of it	0.22
12	12	say: it might have been worse	0.29
13	13	experience unrest	0.14
14	14	talk to friends and family	0.22
15	15	seek diversion	0.16
16	16	avoid nose breathing	0.17
17	17	try to adapt to situation	0.69

### 3.5 Table 5.3

Get the data, and dichotomize the scores, see previous example

Automated item selection algorithm with different values for the lower bound.

```
> aisp(X, lowerbound = 0, verbose = FALSE)
> aisp(X, lowerbound = 0.05, verbose = FALSE)
> aisp(X, lowerbound = 0.1, verbose = FALSE)
```

Make the table (advanced R code)

```
> lower.bound <- seq(0, 0.6, by = 0.05)
> scaling.results <- matrix(NA, length(lower.bound),
+   ncol(X))
> for (i in 1:length(lower.bound)) scaling.results[i,
+   ] <- aisp(X, lowerbound = lower.bound[i],
+   verbose = FALSE)
> equal <- function(x, n) which(x == n)
> scale.1 <- sapply(apply(scaling.results, 1, "equal",
+   1), paste, collapse = " ")
> scale.2 <- sapply(apply(scaling.results, 1, "equal",
+   2), paste, collapse = " ")
> scale.3 <- sapply(apply(scaling.results, 1, "equal",
+   3), paste, collapse = " ")
> scale.4 <- sapply(apply(scaling.results, 1, "equal",
```



```
> scale.35 <- aisp(X, lowerbound = 0.35)
```

Make the table (advanced R code)

```
> scale.30 <- aisp(X, lowerbound = 0.3, verbose = F)
> max.scale <- max(scale.30)
> Table.5.4.left <- data.frame()
> for (i in 1:max.scale) {
+   max.item <- max(length(scale.30[scale.30 ==
+     i]))
+   Scale <- c(i, rep("", max.item - 1))
+   Item.30 <- which(scale.30 == i)
+   Hi.30 <- round(coefH(X[, scale.30 == i], FALSE)$Hi,
+     2)
+   H.30 <- c(rep("", max.item - 1), round(coefH(X[,
+     scale.30 == i], FALSE)$H, 2))
+   Table.5.4.left <- rbind(Table.5.4.left, data.frame(Scale = Scale,
+     Item = Item.30, Hi = Hi.30, H = H.30),
+     c("", "", "", ""))
+ }
> rownames(Table.5.4.left) <- NULL
> Table.5.4.left
```

	Scale	Item	Hi	H
1	1	3	0.57	
2		5	0.51	
3		6	0.3	
4		7	0.53	
5		9	0.5	
6		11	0.45	0.47
7				
8	2	1	0.52	
9		2	0.61	
10		4	0.61	
11		13	0.3	0.55
12				
13	3	8	0.41	
14		15	0.38	
15		17	0.49	0.42
16				
17	4	10	0.47	
18		12	0.47	0.47
19				

```

> scale.35 <- aisp(X, lowerbound = 0.35, verbose = F)
> max.scale <- max(scale.35)
> Table.5.4.right <- data.frame()
> for (i in 1:max.scale) {
+   max.item <- max(length(scale.35[scale.35 ==
+     i]))
+   Scale <- c(i, rep("", max.item - 1))
+   Item.35 <- which(scale.35 == i)
+   Hi.35 <- round(coefH(X[, scale.35 == i], FALSE)$Hi,
+     2)
+   H.35 <- c(rep("", max.item - 1), round(coefH(X[,
+     scale.35 == i], FALSE)$H, 2))
+   Table.5.4.right <- rbind(Table.5.4.right,
+     data.frame(Scale = Scale, Item = Item.35,
+       Hi = Hi.35, H = H.35), c("", "", "",
+       ""))
+ }
> rownames(Table.5.4.right) <- NULL
> Table.5.4.right

```

	Scale	Item	Hi	H
1		1	3 0.6	
2			5 0.53	
3			7 0.6	
4			9 0.63	
5			11 0.49 0.55	
6				
7	2	1	0.54	
8		2	0.68	
9		4	0.67 0.64	
10				
11	3	8	0.41	
12		15	0.38	
13		17	0.49 0.42	
14				
15	4	13	0.53	
16		14	0.53 0.53	
17				
18	5	10	0.47	
19		12	0.47 0.47	
20				

### 3.7 Table 6.1

Get the data. The two pseudo task. Item 2 (column 3) and item 4 (column 5) were not considered. Also, the first column (Group) is removed from the data. Tasks 3 and 4 (items 5 and 9) were investigated in detail. This is the item pair number 21.

```
> library(mokken)
> data(transreas)
> X <- transreas[, -c(1, 3, 5)]
> check.restscore(X, minsize = 2)$results[[21]]

[[1]]
[[1]]$FIRST.ITEM
[1] "T04W"

[[1]]$SECOND.ITEM
[1] "T03W"

$SUMMARY.MATRIX
      Group Lo Hi   N   E(X3)   E(X7) P(X3>=1)
[1,]      1  1  1    2 0.5000000 0.0000000 0.5000000
[2,]      2  2  2    4 0.5000000 0.0000000 0.5000000
[3,]      3  3  3    8 0.7500000 0.3750000 0.7500000
[4,]      4  4  4   27 0.7407407 0.6296296 0.7407407
[5,]      5  5  5   75 0.7600000 0.8266667 0.7600000
[6,]      6  6  6  127 0.8110236 0.9370079 0.8110236
[7,]      7  7  7  117 0.7863248 0.9487179 0.7863248
[8,]      8  8  8   65 0.8000000 0.9846154 0.8000000
      P(X7>=1)
[1,] 0.0000000
[2,] 0.0000000
[3,] 0.3750000
[4,] 0.6296296
[5,] 0.8266667
[6,] 0.9370079
[7,] 0.9487179
[8,] 0.9846154

$VIOLATION.MATRIX
      #ac #vi #vi/#ac maxvi      sum
P(X3>=1) P(X7>=1)    7    4 0.5714286    0.5 1.486111
Total          7    4 0.5714286    0.5 1.486111
```

		sum/#ac	zmax	#zsig
P(X3>=1)	P(X7>=1)	0.2123016	0.8913385	0
Total		0.2123016	0.8913385	0

#### \$Z.Scores

		Rest-score 1-1	Rest-score 2-2
P(X3>=1)	P(X7>=1)	0	0.69194
		Rest-score 3-3	Rest-score 4-4
P(X3>=1)	P(X7>=1)	0.8913385	0.5142975
		Rest-score 5-5	Rest-score 6-6
P(X3>=1)	P(X7>=1)	0	0
		Rest-score 7-7	Rest-score 8-8
P(X3>=1)	P(X7>=1)	0	0

> check.restscore(X, minsize = 40)\$results[[21]]

[[1]]

[[1]]\$FIRST.ITEM

[1] "T04W"

[[1]]\$SECOND.ITEM

[1] "T03W"

#### \$SUMMARY.MATRIX

	Group	Lo	Hi	N	E(X3)	E(X7)	P(X3>=1)
[1,]	1	1	4	41	0.7073171	0.4878049	0.7073171
[2,]	2	5	5	75	0.7600000	0.8266667	0.7600000
[3,]	3	6	6	127	0.8110236	0.9370079	0.8110236
[4,]	4	7	7	117	0.7863248	0.9487179	0.7863248
[5,]	5	8	8	65	0.8000000	0.9846154	0.8000000

P(X7>=1)

[1,]	0.4878049
[2,]	0.8266667
[3,]	0.9370079
[4,]	0.9487179
[5,]	0.9846154

#### \$VIOLATION.MATRIX

		#ac	#vi	#vi/#ac	maxvi	sum
P(X3>=1)	P(X7>=1)	4	1	0.25	0.2195122	0.2195122
Total		4	1	0.25	0.2195122	0.2195122

		sum/#ac	zmax	#zsig
P(X3>=1)	P(X7>=1)	0.05487805	1.679342	1

```
Total          0.05487805 1.679342      1
```

```
$Z.Scores
```

```

Rest-score 1-4 Rest-score 5-5
P(X3>=1) P(X7>=1)      1.679342      0
Rest-score 6-6 Rest-score 7-7
P(X3>=1) P(X7>=1)      0      0
Rest-score 8-8
P(X3>=1) P(X7>=1)      0
```

```

> plot(check.restscore(X, minsize = 2), item.pairs = 21)
> plot(check.restscore(X, minsize = 40), item.pairs = 21)
> R <- apply(X[, -c(3, 7)], 1, sum)
> table(X[, 3], X[, 7], R)
```

```
, , R = 1
```

```

  0  1
0  1  0
1  1  0
```

```
, , R = 2
```

```

  0  1
0  2  0
1  2  0
```

```
, , R = 3
```

```

  0  1
0  1  1
1  4  2
```

```
, , R = 4
```

```

  0  1
0  1  6
1  9 11
```

```
, , R = 5
```

```

      0  1
0  0 18
1 13 44

, , R = 6

```

```

      0  1
0  1 23
1  7 96

, , R = 7

```

```

      0  1
0  2 23
1  4 88

, , R = 8

```

```

      0  1
0  0 13
1  1 51

```

```

> as.numeric(table(X[, 3][R < 5], X[, 7][R < 5]))

[1]  5 16  7 13

```

### 3.8 Table 6.2

Get the data. The two pseudo task. Item 2 (column 3) and item 4 (column 5) were not considered. Also, the first column (Group) is removed from the data.

```

> library(mokken)
> data(transreas)
> X <- transreas[, -c(1, 3, 5)]
> Task <- c(9, 10, 4, 5, 2, 7, 3, 1, 8, 6)
> ppp <- check.pmatrix(X)$results$Ppp
> dimnames(ppp) <- list(Task, Task)
> round(ppp, 2)

```



	9	10	4	5	2	7	3	1	8	6
9	NA	0.22	0.23	0.25	0.24	0.28	0.28	0.29	0.30	0.30
10	0.22	NA	0.39	0.44	0.42	0.48	0.49	0.51	0.51	0.52
4	0.23	0.39	NA	0.64	0.69	0.67	0.69	0.73	0.76	0.76
5	0.25	0.44	0.64	NA	0.66	0.69	0.73	0.77	0.78	0.79
2	0.24	0.42	0.69	0.66	NA	0.68	0.72	0.76	0.78	0.80
7	0.28	0.48	0.67	0.69	0.68	NA	0.79	0.82	0.83	0.84
3	0.28	0.49	0.69	0.73	0.72	0.79	NA	0.86	0.88	0.88
1	0.29	0.51	0.73	0.77	0.76	0.82	0.86	NA	0.92	0.93
8	0.30	0.51	0.76	0.78	0.78	0.83	0.88	0.92	NA	0.96
6	0.30	0.52	0.76	0.79	0.80	0.84	0.88	0.93	0.96	NA

```

> library(mokken)
> data(transreas)
> X <- transreas[, -c(1, 3, 5)]
> Task <- c(9, 10, 4, 5, 2, 7, 3, 1, 8, 6)
> pmm <- check.pmatrix(X)$results$Pmm
> dimnames(pmm) <- list(Task, Task)
> round(pmm, 2)

```

	9	10	4	5	2	7	3	1	8	6
9	NA	0.40	0.14	0.15	0.13	0.13	0.10	0.05	0.03	0.02
10	0.40	NA	0.08	0.12	0.09	0.12	0.09	0.04	0.03	0.02
4	0.14	0.08	NA	0.05	0.09	0.04	0.02	0.01	0.01	0.01
5	0.15	0.12	0.05	NA	0.04	0.04	0.04	0.02	0.01	0.01
2	0.13	0.09	0.09	0.04	NA	0.03	0.02	0.01	0.01	0.01
7	0.13	0.12	0.04	0.04	0.03	NA	0.06	0.03	0.02	0.02
3	0.10	0.09	0.02	0.04	0.02	0.06	NA	0.04	0.02	0.02
1	0.05	0.04	0.01	0.02	0.01	0.03	0.04	NA	0.01	0.01
8	0.03	0.03	0.01	0.01	0.01	0.02	0.02	0.01	NA	0.01
6	0.02	0.02	0.01	0.01	0.01	0.02	0.02	0.01	0.01	NA

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